The Pearson Institute Discussion Paper No. 31

Factional conflict and territorial rents

Ethan Bueno de Mesquita

2016

Working paper

THE PEARSON INSTITUTE
FOR THE STUDY AND RESOLUTION OF GLOBAL CONFLICTS

Factional Conflict and Territorial Rents *

Ethan Bueno de Mesquita[†]

May 27, 2016

Abstract

I study a model of factional conflict over territories from which rents are endogenously generated through market power. Factionalization leads to more frequent, but less intense, conflict. As a result, factionalization is associated with a decrease in both the variability of violence and the stability of the configuration of territorial control, but has a non-monotone relationship to expected violence. Consistent with standard intuitions, changes to economic conditions that increase market power or market size at all territories lead to a positive association between rents and conflict. However, contrary to these same intuitions, changes in local economic conditions at a territory under dispute lead to a negative association between rents and conflict. The local comparative statics facilitate a theoretical exploration of the sign and magnitude of the bias associated with standard empirical strategies.

*I have received helpful comments from Dan Alexander, Scott Ashworth, Dan Black, Bruce Bueno de Mesquita, Oeindrila Dube, Catherine Hafer, Emir Kamenica, Pablo Montagnes, Alejandro Poiré Romero, Jesse Shapiro, Stephane Wolton, and seminar audiences at Chicago, CIDE, Georgetown, and Michigan. This research was supported by the Office of Naval Research under ONR Grant Number N00014-13-1-0138. Any opinions, findings, or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the Office of Naval Research.

†Harris School of Public Policy Studies, University of Chicago, email: bdm@uchicago.edu

In many settings, armed factions compete for control of territory that can be used to extract economic rents.[1]

Some of the most significant factional violence over the last decade in Afghanistan occurred in Helmand Province, a Taliban stronghold and Afghanistan's leading poppy producer. The United Nations Office of Drugs and Crime reports that local factional leaders use control over these territories and transshipment routes to extract economic rents by levying taxes on drug traffickers, poppy farmers, and owners of heroin laboratories.[2]

Over 500 people were murdered in Chicago in 2012. "Most of Chicago's violent crime," according to he head of the DEA for the five-state region that includes Illinois, "comes from gangs trying to maintain control of drug-selling territories."[3] Investigative journalist John Lippert reports that the gangs are motivated precisely by access to territorial rents: "[i]f you want to expand your sales, you have to expand your street corners. You know, you have to physically take street corners, which is a violent act."[4] A similar story of territorial conquest for economic rent extraction is told about violent conflict among Brazil's drug gangs for control of the *favelas* (Lessing, 2013).

From March through June of 2009 violence in the Mexican state of Michoacán more than tripled, reaching an average 67 drug-related homicides per month. The proximate cause was a factional war over territory. Los Zetas, in the midst of splitting from the Gulf Cartel, sought to wrest control over Michoacán and its valuable transshipment routes from La Familia. Such territorial conflicts reached their apex in 2011 and 2012 when over 10,000 people per year lost their lives as a result of the fight between two of the largest Mexican drug trafficking organizations—Los Zetas and the Sinaloa Cartel—for control of transshipment routes ranging from Veracruz, to Guadalajara, to Nuevo Laredo (Rios, 2013).

[1]For theoretical models of conflict over economic rents (though typically not local rents), see, for example, Hirshleifer (1991); Grossman (1999); Hafer (2006); Fearon (2008*a*,*b*); Chassang and Padro i Miguel (2009); Besley and Persson (2011); Dal Bó and Dal Bó (2011). For a very different model that does have conflict over local rents, see Caselli, Morelli and Rohner (2015). For a quite different approach to territorial disputes, see Carter (2010).

[2]United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011.
http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf

[3]John Lippert. "Heroin Pushed on Chicago by Cartel Fueling Gang Murders." *Bloomberg Markets Magazine*. September 16, 2013. http://www.bloomberg.com/news/2013-09-17/heroin-pushed-on-chicago-by-cartel-fueling-gang-murders.html

[4]"Probing Ties Between Mexican Cartel And Chicago's Violence." *National Public Radio*. http://www.npr.org/2013/09/17/223309103/probing-ties-between-mexican-drug-cartel-and-chicagos-violence

Motivated by such conflicts, a recent empirical literature is increasingly interested in the relationship between territorial control, economic rents, and factional violence. (See, for example, Angrist and Kugler (2008); Castillo, Mejia and Restrepo (2013); Mejia and Restrepo (2013); Dube, García-Ponce and Thom (Forthcoming); Dell (Forthcoming); Caselli, Morelli and Rohner (2015).)

I propose a model to investigate these relationships. The model makes three types of contributions. First, it yields testable hypotheses about the effects of factionalization, market power, and market size on conflict outcomes. Second, it allows for a theoretical exploration of standard identification strategies used in the empirical literature. Third, it highlights the conceptual importance of endogenizing the economic returns to territorial conquest.

Factionalization Qualitative accounts and conventional wisdom often suggest that increased factionalization causes an increase in violence.[5] The analysis here finds matters are more subtle.

In the model, increased factionalization is associated with more frequent, but lower-intensity, conflict and less stable territorial control. Overall, expected violence is non-monotone in the number of factions. Three factors drive these relationships. First, factionalization diminishes market power and, thus, decreases incentives for territorial conquest. This accounts for the decrease in conflict intensity. Second, factionalization decreases scare-off (where some faction cedes a territory to another faction) by diminishing the difference between competing factions' returns to territorial conquest. Third, factionalization is associated with the disappearance of "safe territories"—territories that are insulated from attack by virtue of being surrounded entirely by territories controlled by the same faction. These two factors account for the increase in the frequency of conflict and the decrease in the stability of territorial control. The overall non-monotone relationship between factionalization and expected violence is a consequence of the interaction between increased frequency and decreased intensity.

Market Size and Market Power I study two types of comparative statics—global and local—regarding the effects of increased market power and market size. The global

[5]See, for example, Beittel (2013) or Jeremy Garner. Gang factions lead to spike in city violence. *The Chicago Tribune*, October 3, 2012. Available: http://articles.chicagotribune.com/2012-10-03/news/ct-met-street-gang-bloodshed-20121003_1_gang-violence-gangster-disciples-black-p-stones.

comparative statics analyze what happens to the distribution of violence when market conditions change at all territories. Such changes increase both expected violence and the variability of violence.

The comparative statics regarding the effect of variation in local economic conditions on conflict are more interesting. First, local economic shocks at a disputed territory create a negative association between rents and violence, exactly the reverse of the global comparative statics. To see why, consider a shock to the market size surrounding some disputed territory. An increase in local market size increases the marginal costs to raising prices (in terms of foregone demand) for the factions that control surrounding territories. Hence, as local market size at some territory increases, prices at the surrounding territories decrease, which spills over into lower prices at all territories. While this price decline tends to reduce all factions' rents, the rents decrease more slowly for whichever faction ends up with control over the shocked territory, since increased market size also has a direct positive effect on demand at that territory. As a consequence, the returns to territorial conquest are increasing in local market size. Hence, even though the shock decreases all factions' rents, it increases expected violence. (A similar intuition holds in reverse for shocks to local market power.)

Relationship to Empirical Literature The model has at least two implications relevant for empirical scholarship on conflict. The first comes from the contrast between the local and global comparative statics. Often, empirical work assumes that territorial conflict is expected to increase with rents. But that intuition comes from thinking about changes akin to my global comparative statics. The model here finds that exactly the opposite is true for local changes at the territory under dispute. As the empirical literature becomes increasingly concerned with identification, this is precisely the kind of variation being studied.

Second, because the model has economic spillovers, local shocks at one territory affect violence at other territories in subtle ways. In addition to being testable hypotheses in their own right, such results are relevant for thinking about the difference-in-differences identification strategy used in many studies estimating the effect of economic shocks on conflict.[6] While it is obvious (and well known) that if there are spillovers, difference-in-differences is biased, the model goes one step further—allowing empirical and theoretical

[6]See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (Forthcoming); Maystadt and Ecker (2014); Mitra and Ray (2014). A related literature looks at the effect of local development aid on local conflict (e.g., Berman, Shapiro and Felter, 2011; Crost, Felter and Johnston, 2014).

research to constructively engage by using theory to explore the sign and magnitude of the resulting bias.

In the case of shocks to market size, the model predicts that difference-in-differences yields overestimates—the shock increases violence at the shocked territory and decreases violence at other territories. Excluding the nearest neighbor from the control territories reduces the bias, but does not eliminate it. In the case of shocks to local market power, matters are more complicated. If the nearest neighboring territory is used as the control, the model predicts that both the sign and magnitude of the bias depend on the magnitude of the shock and are, thus, probably unknowable by the empirical researcher. Excluding the nearest neighbor from the control territories can increase or decrease the bias. But at least now the sign of the bias is known. Unfortunately, the bias is such that, again, difference-in-differences overestimates the effect size.

Relationship to the Theoretical Literature The theoretical conflict literature is vast and I do not attempt to summarize it here. But it is worth noting that, in my model, all of the predicted relationships are driven by the fact that the value of territorial conquest is determined endogenously by future economic behavior, which, in turn, depends on market power, market size, and the number of factions. Hence, the model highlights, in one setting, the value of endogenizing the economic returns to conflict for understanding how conflict plays out.[7]

1 The Model

There are six fixed territories, labeled $A - F$, located at equal intervals on the perimeter of a circle.[8] The territories are arrayed in alphabetical order (so territory F is contiguous with territories A and E). The territories are controlled by factions. I consider variants of the game with between two and six factions. Let $\mathcal{F}$ be the set of factions. A population of mass N is located uniformly on the perimeter of the circle.

The game is played as follows.

(i) At the beginning of the game, there is some configuration of factional control of the territories described by a partition of $\{A, B, C, D, E, F\}$.

[7] For models that consider other aspects of the two-way relationship between economic and conflict outcomes, see, Fearon (2008*b*); Besley and Persson (2010); Rohner, Thoenig and Zilibotti (2013).

[8] Six territories is the smallest number needed for the comparisons in Section 4.

(ii) Nature chooses one territory to become *vulnerable*. Any faction, $i \in \mathcal{F}$, that controls either the vulnerable territory or a territory contiguous with it chooses an amount, $a_i \in \mathbb{R}_+$, to invest in fighting for control of the vulnerable territory.

(iii) At the end of the conflict either the territory is still controlled by its original owner or has changed hands. Factions then set prices for the single good traded in the economy. A faction can set a different price at each territory it controls. The price at territory j is $p_j \in [0, 1]$.

(iv) Each population member decides whether and from which territory to buy the good.

Conflict is modeled as an all-pay auction (Krishna and Morgan, 1997; Epstein and Gang, 2007). Call the initial holder of a vulnerable territory the *defender* and all factions with contiguous territories *attackers*. If one of the factions involved in fighting invests strictly more than any other faction, it wins the territory. If the defender is involved in a tie, she wins. If two attackers are involved in a tie, they win with equal probability.[9]

Each population member gets a benefit of 1 from consuming the good. Population members bear linear transportation costs, $t \in (0, 1]$. If a population member buys the good for price p from a territory at distance x from her location, her payoff is $1 - p - tx$. If she doesn't buy the good, her payoff is zero.

The factions bear costs for investing in conflict and make profits from selling the good. If a faction makes revenues r and invests a in conflict, its payoff is $r - a$.

I will primarily be interested in the amount of *observed violence*. Say that violence is observed if at least two factions make a positive investment. If violence is observed, it is the sum of the investments:

$$v = \begin{cases} \sum_{i \in \mathcal{F}} a_i & \text{if } \big| \{i \in \mathcal{F} : a_i > 0\} \big| \geq 2 \\ 0 & \text{else,} \end{cases}$$

where a_i is constrained to be zero for factions that are neither attackers nor the defender.

The solution concept is subgame perfect Nash equilibrium.

1.1 Comments on the Model

Before turning to the analysis, I briefly discuss some assumptions and matters of interpretation.

[9] Since ties never occur in equilibrium, the tie breaking rule is irrelevant.

Most important is the interpretation of transportation costs which, because they create imperfect competition, are the source of market power in the model. In the case of drug gangs that control street corners in the United States or *favelas* in Brazil, transportation costs can be understood as a model of consumers' search and travel costs for finding alternative suppliers. Afghan factions often tax travel on roads they control and charge for protection services.[10] The associated market power depends on the availability of alternative routes, which are reasonably modeled as transportation costs. For Mexican drug transshipment, and in some other potential applications, market power derives from sources that are perhaps more distant from transportation costs. The model maps less cleanly onto such cases, but even so may provide some insight if we think of transportation costs as a metaphor for market power more generally. Of course, one could also ask a variety of interesting questions about the spillover of violence onto transportation costs or about factions' strategic use of violence to manipulate market power. Those questions are left for future research, as are potentially important dynamic considerations.

It is also worth highlighting a few assumptions. First, because total rents are increasing in market concentration, the factions would benefit from forming a cartel. Hence, the model implicitly assumes a commitment problem preventing such agreements (Fearon, 1995; Powell, 2004).

Second, violence has no negative welfare consequences for consumers (Besley and Mueller, 2012). Since I do not provide a welfare analysis, no results would be changed by allowing such externalities.

Third, only one territory is vulnerable and only contiguous factions can attack that territory. These assumptions aid with tractability, but also capture a substantively plausible intuition. As factions consolidate, more territories are located in the interior of factions' areas of control. These internal territories are harder to attack and, thus, consolidation reduces opportunities for conflict.[11] I point out when this effect plays a role in the analysis.

Finally, a faction bears the costs of investment in conflict even if the other factions do not

[10] United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011.
http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf

[11] See Papachristos, Hureau and Braga (2013) and Dell (Forthcoming) for evidence from sub-state conflicts on the lower frequency of attacks against non-contiguous and interior territories, respectively. Of course, a long literature on inter-state conflict consistently finds that contiguity is one of the best predictors of conflict (e.g., Bremer, 1992; Vasquez, 1995; Reed and Chiba, 2010).

invest (ceding the territory). Since preparing for conflict involves converting resources into training and weapons, factions surely bear some costs in such circumstances. A somewhat more satisfying assumption might be that these costs are lower when no actual fighting occurs. However, the benefits of such an assumption, in terms of verisimilitude, come at a significant cost in tractability.

2 Conflict for General Incremental Returns

A faction deciding how much to invest in fighting for the vulnerable territory compares its expected payoff economic rents should it win versus lose the fight. Label a configuration of territorial control and vulnerability ξ. Call the difference in faction j's expected equilibrium rents should it win versus lose its *incremental return to winning* at ξ, IR_j^{ξ}.

At most three factions can be involved in conflict. As we will see later, in the configurations of interest, it turns out that, even when three factions can fight, the defender's incremental return is strictly lower than the attackers'. As such, the following results from the literature on all-pay auctions are key:

Theorem 2.1 *(Hillman and Riley, 1989; Baye, Kovenock and De Vries, 1996) In an all-pay auction with linear costs, let IR_i be player i's expected incremental return from winning the auction instead of losing the auction. If either there are two players with $\mathrm{IR}_1 \geq \mathrm{IR}_2$ or there are three players with $\mathrm{IR}_1 \geq \mathrm{IR}_2 > \mathrm{IR}_3$, then there is a unique equilibrium. In it, Player 1 bids the realization of a random variable drawn from the uniform distribution on $[0, \mathrm{IR}_2]$ and Player 2 bids 0 with probability $\frac{\mathrm{IR}_1 - \mathrm{IR}_2}{\mathrm{IR}_1}$ and with the complementary probability bids the realizations of an independent random variable drawn from the uniform distribution on $[0, \mathrm{IR}_2]$. Player 3 (if she exists) bids zero.*

There is a subtlety associated with calculating the incremental returns in my model that does not exist in the standard auction setting. In an all-pay auction, a player's incremental return is simply the value of the asset. Here, economic rents are sensitive to the configuration of territorial control. Hence, in a conflict potentially involving three factions, a faction's expected payoff should it lose depends on its beliefs about the likelihood that each other faction wins, which depends on those factions' strategies. I attend to this issue when characterizing equilibrium in Section 4. Here, I build intuitions for the case where only two factions actively fight, which turns out to be the case of interest.

Suppose that either two or three factions $(1, 2$ and $3)$ can fight over a vulnerable territory and that $\mathrm{IR}_1^{\xi} \geq \mathrm{IR}_2^{\xi} (> \mathrm{IR}_3^{\xi})$. Theorem 2.1 indicates that faction 3 cedes the territory for certain and faction 2 cedes the territory with positive probability if $\text{IR}_2^\xi < \text{IR}_1^\xi$. So a conflict does not always result in observed violence.From an ex ante perspective, the amount of observed violence is a random variable. With probability $\frac{\text{IR}_1^\xi - \text{IR}_2^\xi}{\text{IR}_1^\xi}$, faction 2 cedes and v takes the value 0. With complementary probability, v is the sum of two uniform random variables on $[0, \text{IR}_2^\xi]$ and, so, has a symmetric triangular distribution on $[0, 2\text{IR}_2^\xi]$. Hence, v has a CDF given by

$$\Phi^\xi(v) = \begin{cases} 1 - \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi} + \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}\left(\frac{v^2}{2(\text{IR}_2^\xi)^2}\right) & \text{if } v \in [0, \text{IR}_2^\xi] \\ \\ 1 - \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi} + \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}\left(1 - \frac{(2\text{IR}_2^\xi - v)^2}{2(\text{IR}_2^\xi)^2}\right) & \text{if } v \in [\text{IR}_2^\xi, 2\text{IR}_2^\xi]. \end{cases} \tag{1}$$

With this, it is straightforward to calculate expected observed violence:

$$E[v|\xi] = \int_0^{2\text{IR}_2^\xi} v \, d\Phi^\xi(v) = \frac{(\text{IR}_2^\xi)^2}{\text{IR}_1^\xi}.$$

Let's unpack the intuition. As faction 1's incremental return increases, faction 1 becomes more willing to invest in conflict. Were it to do so, this would make the second faction unwilling to fight at all, since it would be so likely to lose. But if the first faction is certain the second faction will not fight, then it has no incentive to invest. To maintain equilibrium, as IR_1^ξ increases, faction 1's increased willingness to invest leads faction 2 to cede more often, which establishes equilibrium by decreasing faction 1's incentive to invest. Thus, this *scare-off* effect of an increase in IR_1^ξ tends to reduce the expected amount of observed violence by increasing the probability that the territory is ceded.

An increase in IR_2^ξ has two effects. First, as faction 2's incremental return to winning increases, faction 2 becomes less willing to cede the territory. This *anti-scare-off* effect increases expected observed violence. Second, as faction 2's incremental return increases, faction 2 becomes willing to invest more. This *stakes* effect increases both factions' expected investment and, thus, also increases expected observed violence.

Often some factor will simultaneously increase both IR_1^ξ and IR_2^ξ. Such a change can increase or decrease expected observed violence, depending on the relative effects on the two incremental returns. Note, however, that IR_2^ξ increases expected observed violence through two mechanisms—anti-scare-off and stakes—while IR_1^ξ decreases expected observed violence through only one mechanism—scare-off. Hence, if some factor were to change both incremental returns by similar amounts, the effect on IR_2^ξ would dominate. Indeed, in order for the effect on IR_1^ξ to dominate, it must be more than twice as large. To see this, suppose that both incremental returns are strictly increasing, differentiable functions of some parameter θ. Then expected observed violence is decreasing in θ if and only if:

$$\frac{\partial \mathrm{IR}_2^\xi(\theta)/\partial\theta}{\partial \mathrm{IR}_1^\xi(\theta)/\partial\theta} < \frac{\mathrm{IR}_2^\xi(\theta)}{2\mathrm{IR}_1^\xi(\theta)}. \tag{2}$$

3 Economic Equilibrium

To calculate factions' incremental returns to winning, we need to compute each factions' rents in the economic equilibrium that follows conflict.

Consider two contiguous territories, i and j, charging prices p_i and p_j. A population member located between i and j at distance x from i will purchase from i rather than purchasing from j or staying home if:

$$p_i + tx \leq p_j + t\left(\frac{1}{6} - x\right) \qquad \text{and} \qquad 1 - p_i - tx \geq 0.$$

The population member who is indifferent between purchasing from i and j is located at distance $x^*_{i,j}$ from i, given by:

$$x^*_{i,j} = \frac{1}{12} + \frac{p_j - p_i}{2t}.$$

Plugging this in and rearranging, this population member will purchase if

$$p_i \leq 2 - p_j - \frac{t}{6}. \tag{3}$$

If Condition 3 holds, demand at territory i from population members located between i and j is:[12]

$$D_i(p_i,p_j) = \begin{cases} \frac{N}{6} & \text{if } p_i < p_j - \frac{t}{6} \\ N\left(\frac{1}{12} + \frac{p_j - p_i}{2t}\right) & \text{if } p_i \in \left[p_j - \frac{t}{6}, p_j + \frac{t}{6}\right] \\ 0 & \text{if } p_i > p_j + \frac{t}{6}. \end{cases} \tag{4}$$

I computate the economic equilibrium for all relevant configurations in Appendix D. To fix ideas, I illustrate two cases. I adopt the notation that territory $i+1$ is the territory one letter higher in the alphabet than i, except in the case of F, where $F+1 = A$.

First suppose there are two symmetric factions, one controlling territories A, B, C and

[12] Appendix D shows that Condition 3 always holds in equilibrium.

the other controlling territories D, E, F. (I notate this $3, 3$, since there are two factions, each controlling 3 territories.) If demand is characterized by Equation 4 at some vector of prices, the factions' rents are:

$$\sum_{i=A}^{C} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] \qquad \text{and} \qquad \sum_{i=D}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

From the first-order conditions, equilibrium prices are

$$p_A^* = p_C^* = p_D^* = p_F^* = \frac{t}{2} \qquad \text{and} \qquad p_B^* = p_E^* = \frac{7t}{12}.$$

Notice several facts. First, prices are higher at interior territories (B and E), reflecting greater market power. Second, for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ and each consumer purchases from one of the two territories to which she is closest, so demand is described by Equation 4. Equilibrium rents for each faction are

$$u^{3,3} = \frac{37Nt}{144}.$$

Now suppose there are two factions, one controlling territories A, B, C, D and the other controlling territories E, F (I denote this $4, 2$). If demand is characterized by Equation 4 at some vector of prices, the large and small factions' rents, respectively, are:

$$\sum_{i=A}^{D} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] \qquad \text{and} \qquad \sum_{i=E}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

From the first-order conditions, equilibrium prices are

$$p_A^* = p_D^* = \frac{5t}{9} \qquad p_B^* = p_C^* = \frac{13t}{18} \qquad p_E^* = p_F^* = \frac{4t}{9}.$$

Again, for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ and each consumer purchases from one of the two territories to which she is closest, so demand is described by Equation 4. The large and small factions' equilibrium rents, respectively, are:

$$u^{\mathbf{4},2} = \frac{109Nt}{324} \qquad u^{4,\mathbf{2}} = \frac{16Nt}{81}.$$

A few points are worth emphasizing. First, prices again are higher at interior territories (B and C). Second, the large faction charges higher prices than the small faction at border territories, reflecting its greater market power. Third, because consolidation leads to higher prices, total rents are higher with unequal factions than with equal factions:

$$u^{\mathbf{4},2} + u^{4,\mathbf{2}} = \frac{173Nt}{324} > \frac{74Nt}{144} = u^{3,3} + u^{3,3}. \tag{5}$$

Table 3 summarizes factions' rents for all relevant configurations of territorial control (details in Appendix D). Configurations are notated according to the number of territories controlled by each faction. So, for example, $3, 2, 1$ is a configuration with three factions controlling three, two, and one territories, respectively. For payoffs, the relevant faction's number is in bold, so those factions' rents are $u^{\mathbf{3},2,1}$, $u^{3,\mathbf{2},1}$, and $u^{3,2,\mathbf{1}}$, respectively.

Configuration	Highest Payoff	2nd Highest Payoff	3rd Highest Payoff
$1, 1, 1, 1, 1, 1$	$u^{1,1,1,1,1,1} = \frac{Nt}{36}$		
$2, 1, 1, 1, 1$	$u^{\mathbf{2},1,1,1,1} = \frac{145Nt}{2166}$	$u^{2,\mathbf{1},1,1,1} = \frac{40Nt}{1083}$	$u^{2,1,\mathbf{1},1,1} = \frac{100Nt}{3249}$
$2, 2, 2$	$u^{2,2,2} = \frac{Nt}{9}$		
$3, 2, 1$	$u^{\mathbf{3},2,1} = \frac{447{,}343Nt}{2{,}643{,}878}$	$u^{3,\mathbf{2},1} = \frac{298{,}831Nt}{2{,}643{,}876}$	$u^{3,2,\mathbf{1}} = \frac{5041Nt}{73{,}441}$
$3, 3$	$u^{3,3} = \frac{37Nt}{144}$		
$4, 2$	$u^{\mathbf{4},2} = \frac{109Nt}{324}$	$u^{4,\mathbf{2}} = \frac{16Nt}{81}$	

Table 3.1: Rents associated with different configurations of territorial control.

4 Number of Factions and Conflict

I now query the effect of the number of factions on violent outcomes. In order to hold all else equal while changing factionalization, I compare *symmetric-connected configurations*. That is, I consider a configuration of territorial control with six factions each controlling one territory, a configuration with three factions each controlling two contiguous territories, and a configuration with two factions each controling three contiguous territories.

Six Symmetric-Connected Factions If there are six factions and the defender wins the conflict, the outcome is the status quo. If the defender loses the conflict, then it is eliminated. Hence, the defender's incremental return is

$$\mathrm{IR}_{\mathrm{def}}^{1,1,1,1,1,1} = u^{1,1,1,1,1,1} - 0.$$

There are two attackers. If an attacker faction, j, wins the conflict, it becomes the large faction in a five faction configuration. If it loses the conflict, then either the defender wins (and j remains in a six faction configuration) or the other attacker wins (and j becomes a small faction bordering the large faction in a five faction configuration). Let π be j's belief about the probability that the defender invests more than the other attacker. Then faction j's incremental return is

$$\mathrm{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi) = u^{\mathbf{2},1,1,1,1} - \pi u^{1,1,1,1,1,1} - (1-\pi)u^{2,\mathbf{1},1,1,1}.$$

Lemma 4.1 shows that the two attackers have strictly higher incremental returns to winning than does the defender.

Lemma 4.1 $\mathrm{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi) > \mathrm{IR}_{\text{def}}^{1,1,1,1,1,1}$ *for any* $\pi \in [0,1]$.

Proof. See Appendix A. ■

From Theorem 2.1, given that the two attackers' incremental returns are larger than the defender's, the defender invests zero and loses for certain, which implies that $\pi = 0$ in equilibrium. At $\pi = 0$, we have

$$\mathrm{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = u^{\mathbf{2},1,1,1,1} - u^{2,\mathbf{1},1,1,1} = \frac{65Nt}{2166}.$$

Given this, Theorem 2.1 and Equation 1 immediately imply the following:

Proposition 4.1 *When the initial configuration involves six factions, regardless of which territory becomes vulnerable, equilibrium play at the conflict stage is as follows:*

- *The defender invests zero and loses for certain.*
- *The attackers' investments are drawn independently from a uniform distribution on* $\left[0, \frac{65Nt}{2166}\right]$.

The ex ante distribution of observed violence is as in Equation 1, with $\mathrm{IR}_1^{1,1,1,1,1,1} = \mathrm{IR}_2^{1,1,1,1,1,1} = \frac{65Nt}{2166}$.

Three Symmetric-Connected Factions If there are three symmetric-connected factions, the defender's incremental return is

$$\mathrm{IR}_{\text{def}}^{2,2,2} = u^{2,2,2} - u^{3,2,\mathbf{1}} = \frac{28,072Nt}{660,969}$$

and the lone attacker's incremental return is

$$\mathrm{IR}_{\text{att}}^{2,2,2} = u^{\mathbf{3},2,1} - u^{2,2,2} = \frac{51,193Nt}{881,292}.$$

Given this, Theorem 2.1 and Equation 1 immediately imply the following:

Proposition 4.2 *When the initial configuration involves three symmetric-connected factions, regardless of which territory becomes vulnerable, equilibrium at the conflict stage is:*

- *With probability $\frac{112,288}{153,579}$, the defender's investment is drawn from a uniform distribution on $\left[0, \frac{28,072Nt}{660,969}\right]$ and with complementary probability the defender invests zero.*
- *The attacker's investment is drawn independently from a uniform distribution on $\left[0, \frac{28,072Nt}{660,969}\right]$.*

The ex ante distribution of observed violence is as in Equation 1, with $\mathrm{IR}_1^{2,2,2} = \mathrm{IR}_{\text{att}}^{2,2,2} = \frac{51,193Nt}{881,292}$ *and* $\mathrm{IR}_2^{2,2,2} = \mathrm{IR}_{\text{def}}^{2,2,2} = \frac{28,072Nt}{660,969}$.

Two Symmetric-Connected Factions If there are two symmetric-connected factions, there are two cases: the vulnerable territory is on a border or is interior. In the latter case, there is no conflict. So consider the former.

The defender's incremental return is

$$\mathrm{IR}_{\text{def}}^{3,3} = u^{3,3} - u^{4,\mathbf{2}} = \frac{77Nt}{1296},$$

while the attacker's incremental return is

$$\mathrm{IR}_{\text{att}}^{3,3} = u^{\mathbf{4},2} - u^{3,3} = \frac{103Nt}{1296}.$$

Given this, conditional on a border territory being vulnerable, the equilibrium follows from Theorem 2.1. Hence, the ex ante distribution of observed violence is a mixture of an atom on zero with probability mass $\frac{1}{3}$ (the probability an interior territory is vulnerable) and the distribution described in Equation 1 with probability $\frac{2}{3}$, which implies the following:

Proposition 4.3 *When the initial configuration involves two symmetric-connected factions, if the vulnerable territory is interior, there is no conflict. If the vulnerable territory is a border territory, then equilibrium play at the conflict stage is as follows:*

- *With probability $\frac{77}{103}$ the defender's investment is drawn from a uniform distribution on $\left[0, \frac{77Nt}{1296}\right]$ and with complementary probability the defender invests zero.*

- *The attacker's investment is drawn independently from a uniform distribution on $\left[0, \frac{77Nt}{1296}\right]$.*

Consequently, the ex ante distribution of observed violence is given by the following CDF:

$$\Phi^{3,3}(v) = \begin{cases} \frac{1}{3} + \frac{2}{3} \cdot \frac{26}{103} + \frac{2}{3} \cdot \frac{77}{103} \left(\frac{1296^2 v^2}{77 \cdot 154 N^2 t^2} \right) & \textit{if } v \in \left[0, \frac{77Nt}{1296}\right] \\ \frac{1}{3} + \frac{2}{3} \cdot \frac{26}{103} + \frac{2}{3} \cdot \frac{77}{103} \left(1 - \frac{1296^2 \left(\frac{154Nt}{1296} - v\right)^2}{77 \cdot 154 N^2 t^2} \right) & \textit{if } v \in \left[\frac{77Nt}{1296}, \frac{154Nt}{1296}\right]. \end{cases}$$

4.1 Factionalization and Conflict

Given these characterizations, I can now assess the effect of a change in the number of factions on conflict. I decompose the analysis into four parts:

(i) The frequency of observed violence—i.e., the probability that the realization of v is positive.

(ii) The expected intensity of observed violence—i.e., the expectation of v conditional on its realization being positive.

(iii) The variability of observed violence—i.e., the variance of v.

(iv) Expected observed violence—i..e, the unconditional expectation of v.

The results are summarized in Proposition 4.4 below.

Frequency of Violence The frequency of observed violence is determined by how often territory is ceded by all but one faction and the number of safe territories.

With six factions, there are no safe territories and neither attacker cedes. Hence, there is always observed violence. This is not the case when the factions further consolidate. Comparing the configurations with three and two symmetric-connected factions, two things change. On the one hand, Condition 6 below shows that, due to differential changes in attacker's and defender's incremental returns, the defender cedes more often when there are more factions. This tends to increase the frequency of observed violence in more consolidated environments.

$$1 - \frac{\mathrm{IR}^{2,2,2}_{\mathrm{def}}}{\mathrm{IR}^{2,2,2}_{\mathrm{att}}} = \frac{41,291}{153,579} > \frac{26}{103} = 1 - \frac{\mathrm{IR}^{3,3}_{\mathrm{def}}}{\mathrm{IR}^{3,3}_{\mathrm{att}}}. \tag{6}$$

On the other hand, consolidation creates safe territories, reducing opportunities for conflict. These effects pull in competing directions. The net effect, as formalized in Proposition 4.4, is that factionalization is associated with more frequent observed violence.

Intensity of Observed Violence Because consolidation leads to increased economic rents, greater factionalization is associated with smaller incremental returns (conflict is lower stakes) and, thus, a lower expected intensity of observed violence.

Variance of Observed Violence Increased factionalization leads to more frequent, but less intense, observed violence. That is, the more factions, the less likely are both very low (zero) and very high levels of observed violence. Consequently, factionalization is associated with decreased variance.

Expected Observed Violence Given all of these effects, how does the overall expected level of observed violence respond to factionalization? As summarized in Table 4.1, the relationship is non-monotone, reflecting the competing effects of factionalization on intensity and frequency.

To see this, start by noting that, conditional on a border territory being vulnerable, expected observed violence is monotonically decreasing in the number of factions. Because factionalization reduces market power, it reduces both the attacker's and defender's incremental returns to winning. While this generates competing effects the effect on the defender's incremental return dominates (recall Equation 2)—decreasing the incremental returns leads to a decrease in expected observed violence at border territories. But without conditioning on a border territory being vulnerable, there is an additional effect. The consolidation to two factions creates two safe territories so that one-third of the time there is no opportunity for conflict.

Overall, then, equilibrium incentives for investing in conflict decrease as the number of factions increases, but opportunities for conflict increase. As a consequence, from an ex ante perspective, the scenario with only two factions has the lowest expected observed violence even though, conditional on a border territory becoming vulnerable, it has the highest expected observed violence.

Proposition 4.4 *If factions are symmetric-connected, then:*

- *The frequency with which violence is observed is increasing in the number of factions.*

Configuration	IR_1	IR_2	Expected Observed Violence
$1,1,1,1,1,1$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$
$2,2,2$	$\frac{51{,}193}{881{,}292} \approx 0.0581Nt$	$\frac{28{,}072Nt}{660{,}969} \approx 0.0425Nt$	$\frac{3{,}152{,}148{,}736Nt}{101{,}510{,}958{,}051} \approx 0.0311Nt$
$3,3$ (border vulnerable)	$\frac{103Nt}{1296} \approx 0.0795Nt$	$\frac{77Nt}{1296} \approx 0.059Nt$	$\frac{5929Nt}{133{,}488} \approx 0.0444Nt$
$3,3$ (interior vulnerable)	N/A	N/A	0
$3,3$ (ex ante)	N/A	N/A	$\frac{2}{3} \cdot \frac{5929Nt}{133{,}488} \approx 0.0296Nt$

Table 4.1: Expected observed violence as a function of the number of factions.

- *The expected intensity of observed violence—i.e.,* $\mathbb{E}[v|v > 0]$*—is decreasing in the number of factions.*
- *The variance of observed violence is decreasing in the number of factions.*
- *With respect to overall expected observed violence:*
  - *Conditional on a border territory being vulnerable, expected observed violence is decreasing in the number of factions.*
  - *Unconditionally (i.e., allowing for the possibility of interior territories being vulnerable), expected observed violence is non-monotone in the number of factions:*

$$\mathbb{E}[v|2,2,2] > \mathbb{E}[v|1,1,1,1,1,1] > \mathbb{E}[v|3,3].$$

Proof. See Appendix A. ■

4.2 Factionalization and Stability

Because factionalization affects conflict, it affects the stability of the configuration of territorial control. In the most highly factionalized environment (i.e., six factions), the vulnerable territory always changes hands—the defender cedes. In more consolidated configurations, there are two forces at work. First, as was shown in Condition 6, moving from three to two factions decreases scare-off. (This effect is recorded in the second column of Table 4.2, which shows the probability that an attacker wins a conflict, given that a border territory is vulnerable.) Second, consolidation creates safe territories that are not subject to capture. These two effects pull in the same direction. And so, as shown in the third column of Table 4.2 (and stated formally below), factionalization decreases stability.

Configuration	Transition Probability if Border Vulnerable	Overall Transition Probability
$1,1,1,1,1,1$	1	1
$2,2,2$	$\frac{97{,}435}{153{,}579} \approx 0.634$	$\frac{97{,}435}{153{,}579} \approx 0.634$
$3,3$	$\frac{129}{206} \approx 0.626$	$\frac{2}{3} \cdot \frac{129}{206} \approx 0.417$

Table 4.2: Probability a territory changes hands as a function of the number of factions and vulnerability.

Proposition 4.5 *In any symmetric-connected configuration, stability is decreasing in the number of factions.*

5 Global Comparative Statics

In this section I explore global comparative statics—how violent outcomes change as market size or market power change at all territories. In the next section I consider local comparative statics—changes near one particular territory.

In all three symmetric-connected configurations, both factions' incremental returns are linearly increasing in both transportation costs and market size. Hence, a change to either of those parameters has no effect on scare-off, which is determined by the ratio of the incremental returns. Such a change affects observed violence only through the stakes effect. The value of a territory, and thus the size of the stakes effect, is increasing in both t and N. These facts have several implications for the relationship between global market size or market power and conflict outcomes, as recorded in the following result:

Proposition 5.1 *In any symmetric-connected configuration, expected observed violence and the variance of observed violence are increasing in both N and t. Stability is constant in N and t.*

Proof. See Appendix B. ∎

6 Local Comparative Statics

Now I turn to comparative statics for changes that occur locally at one territory. To focus the analysis, I restrict attention to a configuration with two symmetric-connected factions. I ask what happens to observed violence when there are changes to the market size surrounding one particular territory or the transportation costs associated with getting to one particular territory. I consider how observed violence changes when the territory experiencing the market size or transportation cost shock is vulnerable and also what happens when its nearest neighbor controlled by the other faction is vulnerable.

For concreteness, I discuss the case where one faction controls ABC, the other faction controls DEF, and the economic shock is at territory F. I refer to F as the *shocked territory.* I analyze the effects of such shocks when F is vulnerable and when A is vulnerable. In Section 6.3.3, when exploring the relationship of the model to the empirical literature, I also consider what happens when a more distant territory (C or D) is vulnerable.[13]

6.1 Local Market Size

Consider a situation in which the population in the sixth of the circle surrounding territory F increases to $\frac{\eta N}{6}$, for some $\eta \in [1, 2]$, while the population elsewhere stays as it was. (At $\eta = 1$, this is the baseline model.) I first consider the case where F is vulnerable and then turn to the case where A is vulnerable.

6.1.1 Shocked Territory (F) is Vulnerable

To compute the incremental returns, I need the equilibrium rents (as a function of η) in two scenarios: ABC, DEF and $ABCF, DE$.

For a given vector of prices, demand is the same as in Equation 4 at territories B, C, and D but may be changed at A, E, and F. Assuming $p_A \leq 2 - p_j - \frac{t}{6}$, for $j \in \{A, E\}$, demand at territory F from the part of the population between F and j is:

$$D_F(p_F, p_j) = \begin{cases} \frac{(1+\eta)N}{6} & \text{if } p_F \leq p_j - \frac{t}{6} \\ \frac{\eta N}{12} + N\left(\frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j - \frac{t}{6}, p_j\right) \\ \eta N\left(\frac{1}{12} + \frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j, p_j + \frac{t}{6}\right) \\ 0 & \text{if } p_F \geq p_j + \frac{t}{6}. \end{cases} \tag{7}$$

[13]For the case of shocks to market size, the economic and conflict equilibria when territories C or D are vulnerable are characterized in Appendices E.1.4–E.1.5 and F.1–F.2, respectively. For the case of shocks to transportation, the economic and conflict equilibria when territories C or D are vulnerable are characterized in Appendices E.2.4–E.2.5 and F.3–F.4, respectively.

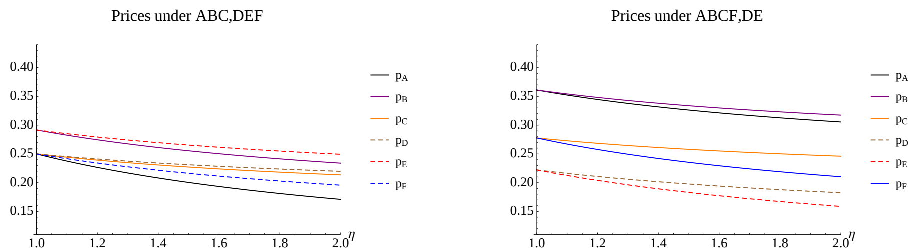

Figure 6.1: Prices as a function of local market size at F for both the ABC, DEF and $ABCF, DE$ configurations.

For territory $j \in \{A, E\}$, demand from the part of the population between j and F is the complement. The economic equilibria are characterized in Appendix E.1.

An increase in local market size has two effects on rents. First, there is a direct effect that tends to increase the rents of the faction that ends up with control of F—for a fixed vector of prices, demand at F increases in η. Second, there is an indirect effect—when local market size around F increases, the marginal cost (in terms of foregone demand) associated with a price increase at A or E increases. Consequently, prices at A and E decrease. Since the economic game has complementarities, this results in price decreases at all territories, as illustrated in Figure 6.1. (For the remainder of the paper, all figures are drawn for the case of $t = 1/2$.) This indirect effect tends to decrease both factions' rents in both configurations.

The left-hand panel of Figure 6.2 illustrates the net effect on rents. Since the faction that does not control territory F at the end of the conflict experiences only the indirect effect, its rents are decreasing in local market size at F. (This is represented by the dashed curves in the figure.) However, for the faction that ends up in control of territory F there are competing effects. Consequently, its rents (represented by the solid curves) are non-monotone in local market size at F. (See Proposition 6.1.)

What does this imply about incremental returns? Because of the direct effect, a faction's rents decrease more slowly in local market size at F if it controls F (indeed, for high enough η, rents can be increasing). Hence, both factions' incremental returns to winning F are increasing in η. This fact is illustrated in the middle panel of Figure 6.2 (and formalized in Proposition 6.1).

Since both factions' incremental returns are increasing in η, there are competing effects on expected observed violence. As shown in Equation 2, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, when conflict is over F, an increase in local

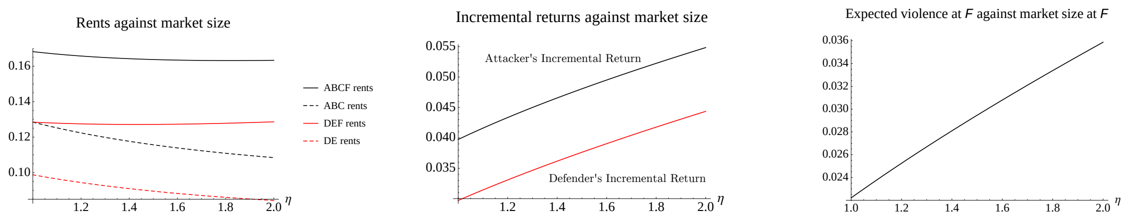

Figure 6.2: Rents, incremental returns, and expected observed violence at F as a function of market size at F.

market size at F increases incremental returns and expected observed violence, even when it decreases all factions' rents. These facts are illustrated in the right-hand panel of Figure 6.2 and formalized in the next result.

Proposition 6.1 *Suppose there are two symmetric-connected factions. Moreover, suppose the population on the sixth of the circle with the vulnerable territory at its center is of mass $\frac{\eta N}{6}$ for some $\eta \in [1, 2]$, while population elsewhere on the circle remains fixed:*

(i) Regardless of what happens at the conflict stage:

- *Rents for the faction that does not end up with control of the vulnerable territory are decreasing in η.*
- *Rents for the faction that ends up with control of the vulnerable territory are decreasing in η at $\eta = 1$, increasing in η at $\eta = 2$, and strictly convex in η.*

(ii) Both factions' incremental returns to winning the conflict over a border territory are increasing in η.

(iii) Expected observed violence is increasing in η.

Proof. See Appendix C.1. ■

The model returns two noteworthy results here. First, an increase in the size of a local market can be associated with a decrease in rents for all factions. Second, even a change in local market size that is associated with a decrease in both factions' rents is associated with an increase in expected observed violence. Hence, contrary to the standard intuitions discussed in Section 5, rents and observed violence need not covary positively when the source of variation is a local economic shock.[14]

[14] Of course, local rents extracted only from territory F are increasing in market size at F, but overall rents need not be.

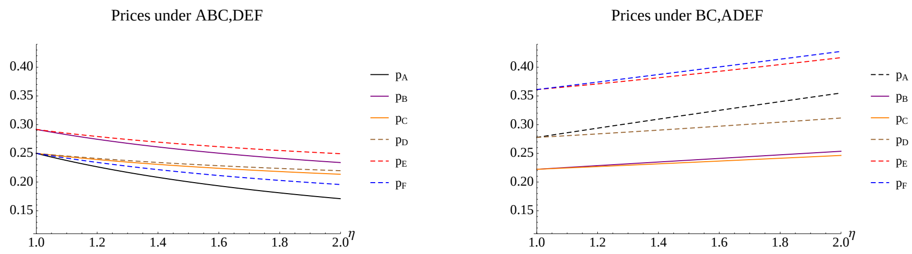

Figure 6.3: Prices as a function of local market size at F for both the ABC, DEF and $BC, ADEF$ configurations.

6.1.2 Shocked Territory's Neighbor (A) is Vulnerable

Continue to consider a shock to market size at F, but suppose territory A is vulnerable. To compute the incremental returns, I need the equilibrium rents (as a function of η) in two scenarios: ABC, DEF and $BC, ADEF$. Again, I characterize the economic equilibria in Appendix E.1.

Figure 6.3 illustrates the key facts. As we've already seen, when different factions control territories A and F (as in the left-hand panel of Figure 6.3), there is an important indirect effect of the population shock—as market size at F increases, it becomes more tempting to lower prices at A, which leads to a diminution in prices at all territories. In contrast, when the faction that controls F also controls A and E (as in the right-hand panel of Figure 6.3) this indirect effect is no longer important because whichever territory the consumers surrounding F buy from, they are customers of the same faction. Consequently, when the attacker wins (so the same faction controls territories A, E, and F), the direct effect dominates—as market size at F increases, the marginal benefit of raising prices at F increases, and both factions' rents increase. (See the $ADEF$ and BC curves in the left-hand panel of Figure 6.4.)

The reversal of the effect of η on prices and rents in the event that the attacker wins (so that A, E, and F are unified) has interesting implications for the incremental returns and observed violence. The defender's rents are decreasing in η if she wins the fight, but increasing in η if she loses the fight. As such, the defender's incremental return is decreasing in η Similarly, the attacker's incremental return is increasing. (See the first two panles of Figure 6.4.) Hence, expected observed violence is decreasing in η. (See the right-hand panel of Figure 6.4.) Further, if η is sufficiently large, the defender's incremental return is negative—she would prefer to cede territory A because doing so leads to increased prices and rents at her remaining territories that compensate for the loss of territory A—so observed

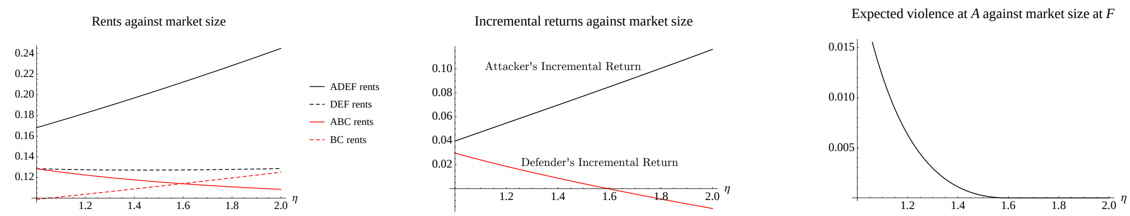

Figure 6.4: Rents, incremental returns, and expected observed violence at territory A as a function of local market size at F.

violence drops to zero. These facts are recorded in the next result.

Proposition 6.2 *Suppose there are two symmetric-connected factions. Moreover, suppose the population on the sixth of the circle with territory j at its center is of mass $\frac{\eta N}{6}$ for some $\eta \in [1, 2]$, while the population elsewhere on the circle remains fixed. If the territory contiguous with j and controlled by the other faction is vulnerable:*

(i) The incremental returns to winning are increasing in η for the attacker and decreasing in η for the defender.

(ii) There is a critical threshold $\hat{\eta} \in (1, 2)$ such that the defender's incremental return is positive for all $\eta \in [1, \hat{\eta})$ and negative for all $\eta \in (\hat{\eta}, 2)$.

(iii) Expected observed violence is strictly decreasing in η for $\eta < \hat{\eta}$ and is zero for $\eta \geq \hat{\eta}$.

Proof. See Appendix C.1. ■

This spillover effect of local market size shocks at F on observed violence at A is a testable hypothesis in its own right. In addition, in Section 6.3 I explore its implications for the empirical literature on the causal effects of economic shocks on conflict.

6.2 Local Transportation Costs

Now consider a situation in which the transportation costs for getting to territory F increase from t to τt for some $\tau \in [1, 2]$. I first consider the case where F is vulnerable and then turn to the case where A is vulnerable.

6.2.1 Shocked Territory (F) is Vulnerable

To compute the incremental returns, I need the economic rents (as a function of τ) in two scenarios: ABC, DEF and $ABCF, DE$.

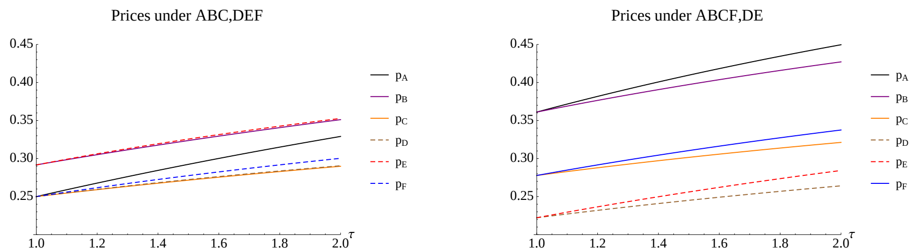

Figure 6.5: Prices as a function of local transportation costs at F.

For a given vector of prices, demand is the same as in Equation 4 at territories B, C, and D but it may be changed at A, E, and F. Fix a vector of prices. As long as $p_F \leq \frac{\tau+1}{\tau} - \frac{p_A}{\tau} - \frac{t}{6\tau}$, for $j \in \{A, E\}$, demand at territory F from the part of the population between F and j is:

$$D_F(p_F, p_j) = \begin{cases} \frac{N}{6} & \text{if } p_F \leq p_j - \frac{\tau t}{6} \\ N\left(\frac{1}{6(\tau+1)} + \frac{p_j - p_F}{t(\tau+1)}\right) & \text{if } p_F \in \left(p_j - \frac{\tau t}{6}, p_j + \frac{\tau t}{6}\right) \\ 0 & \text{if } p_F \geq p_j + \frac{\tau t}{6}. \end{cases} \tag{8}$$

For territory $j \in \{A, E\}$, demand from the population between j and F is the complement. The economic equilibria are characterized in Appendix E.2.

An increase in local transportation costs at F has two effects on rents. First, there is a direct effect that tends to reduce the rents of the faction that ends up with control over F—for a fixed vector of prices, when local transportation costs at F go up, demand at F goes down. Second, there is an indirect effect—when local transportation costs at F go up, the marginal cost (in terms of foregone demand) associated with a price increase at A or E goes down. Consequently, prices at A and E increase. Since the economic game has complementarities, this results in price increases at all territories, as illustrated in Figure 6.5. This indirect effect on prices tends to increase both factions' rents in both configurations. Moreover, as illustrated in the left-hand panel of Figure 6.6 (and formalized in Proposition 6.3), the indirect effect dominates—on net, both factions' rents are increasing in local transportation costs at F in both configurations.

How this effects incremental returns reveals a key intuition. Because of the direct effect, a faction's rents increase more slowly in local transportation costs if it controls F. Hence, although both factions' rents are increasing in τ, their incremental returns to winning F are decreasing in τ. This fact is illustrated in the center panel of Figure 6.6 (and formalized

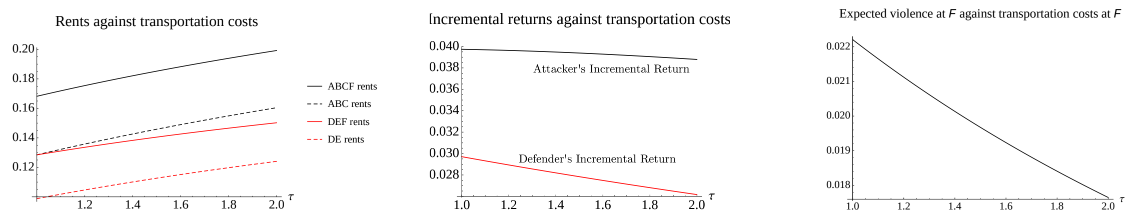

Figure 6.6: Rents, incremental returns, and expected observed violence at F as a function of transportation costs at F.

in Proposition 6.3).

Since both factions' incremental returns are increasing in τ, as shown in Equation 2, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, as illustrated in Figure 6.6, the effect of an increase in local transportation costs at the vulnerable territory is to increase rents, but decrease incremental returns and expected observed violence.

Proposition 6.3 *Suppose there are two symmetric-connected factions. Moreover, suppose the transportation costs associated with the vulnerable territory are τt for some $\tau \in [1, 2]$:*

(i) Regardless of what happens at the conflict stage, both factions' rents at the economic stage are increasing in τ.

(ii) When the vulnerable territory is a border territory, both factions' incremental returns to winning the conflict are decreasing in τ.

(iii) Expected observed violence is decreasing in τ.

Proof. See Appendix C.2. ■

This result is surprising in light of the hypothesized positive association between rents and observed violence which motivates much of the empirical literature. Here, with respect to local transportation costs, exactly the opposite holds—an economic shock that increases overall rents is associated with decreased incremental returns and decreased expected observed violence at the shocked territory.[15]

[15]Similarly to the case of local market size, local rents extracted only from territory F are decreasing in transportation costs at F, but overall rents are not.

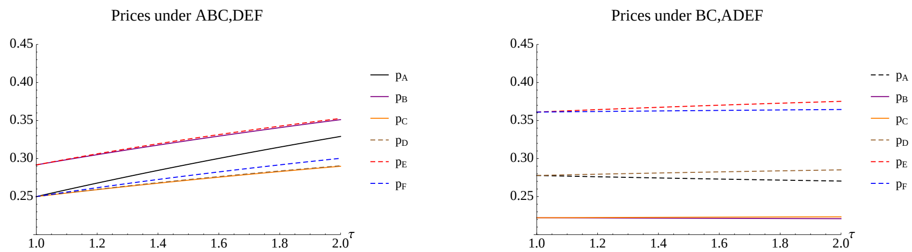

Figure 6.7: Prices as a function of local transportation costs at F.

6.2.2 Shocked Territory's Neighbor (A) is Vulnerable

Continue to consider a shock to transportation costs at F, but now suppose territory A is vulnerable. To compute the incremental returns, I need the economic rents (as a function of τ) in two scenarios: ABC, DEF and $BC, ADEF$. Again, the economic equilibrium is characterized in Appendix E.2.

The key facts are illustrated in Figure 6.7. The intuition when the defender wins is the same as above. But the case where the attacker wins reveals an important contrast between the case where F is vulnerable and the case where A is vulnerable. If the attacker wins control of territory A, prices become significantly less responsive to changes in transportation costs at F. Further, some prices are now decreasing in τ. Both of these facts are intuitive. When the attacking faction wins control of A, territory F no longer faces competition from any territory controlled by the other faction. Consequently, a change to transportation costs at F has little effect because the faction in control of F is relatively unconcerned about whether the consumers surrounding F transact at F or at one of its neighbors.

The fact that prices are relatively unresponsive to transportation costs if the attacker wins ($BC, ADEF$) means that rents are also relatively unresponsive, as illustrated in the left-hand panel of Figure 6.8. Hence, when A is vulnerable, the effect of transportation costs at F on incremental returns is driven by their effect when the defender wins (i.e., ABC, DEF). As we've already seen, in that scenario, both factions' rents are increasing in transportation costs at F. Hence, as illustrated in the center panel of Figure 6.8, the attacker's incremental return is decreasing in transportation costs at F, while the defender's incremental return is increasing in transportation costs at F.

What this implies about expected observed violence depends on whether the attacker's or defender's incremental return is larger. If the attacker's incremental return is larger,

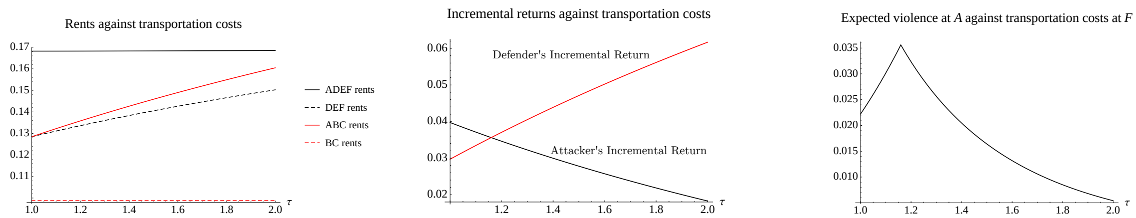

Figure 6.8: Rents, incremental returns, and expected observed violence at territory A as a function of local transportation costs at F.

expected observed violence is increasing in local transportation costs at F. If the defender's incremental return is larger, expected observed violence is decreasing in local transportation costs at F. The center panel of Figure 6.8 shows that the incremental returns cross. This creates a non-monotonicity which is illustrated in the right-hand panel of Figure 6.8 and formalized in the next result.

Proposition 6.4 *Suppose there are two symmetric-connected factions. Let the transportation costs associated with a border territory j be τt for some $\tau \in [1, 2]$. If the territory contiguous with j and controlled by the other faction is vulnerable:*

(i) The incremental returns to winning are increasing in τ for the defender and decreasing in τ for the attacker.

(ii) Expected observed violence is non-monotone in τ. In particular, there is a critical threshold $\hat{\tau} \in (1, 2)$ such that expected observed violence is increasing in τ for $\tau \in [1, \hat{\tau})$ and decreasing in τ for $\tau \in (\hat{\tau}, 2]$.

Proof. See Appendix C.2. ■

This non-monotonic effect of a shock to transportation costs at territory F on observed violence at A is a testable hypothesis in its own right. In addition, in Section 6.3 I explore its implications for the empirical literature on the causal effects of economic shocks on conflict.

6.3 Implications for Empirical Work

These results on the effects of local economic shocks on observed violence have a variety of implications for empirical scholarship. Of course, each comparative static can be interpreted as a testable hypothesis regarding the effects of changes to local market conditions on observed violence. And I have already emphasized the contrast between the global and local comparative statics. But there are also implications that come from thinking about the effects of an economic shock at F on observed violence at other territories.

A common empirical research design uses difference-in-differences to estimate the effect of a local economic shock on observed violence.[16] The treatment effect of interest is the change in observed violence at territory i following an economic shock to territory i at time t. There is concern that other factors that affect observed violence may have changed at the same time that territory i experienced the shock. To isolate the effect of the shock, the researcher studies the change in observed violence at territory i from t to $t+1$ relative to the change in observed violence at nearby territories from t to $t+1$. Under a parallel trends assumption, difference-in-differences identifies the causal effect of the economic shock at territory i on observed violence at territory i.

It is obvious that, in a political economy with economic spillovers like the one modeled here, the parallel trends assumption does not hold. But we can go one step further, asking what the model has to say about the sign and magnitude of the bias. The answer depends on the type, magnitude, and location of the shock.

Above, we calculated observed violence at both F and its nearest neighbor, A, as a function of local shocks at F. So let's start the analysis by asking about the bias of a difference-in-differences estimate if we use the nearest neighbor as the baseline control for F. After studying that quantity, I will investigate what happens to the bias if we use more distant territories as the control.

Since the model has different implications for the two types of shocks, I take them in turn. But it is useful to define some common notation. Write the expected observed violence at territory i given a shock of size σ (which equals τ for shocks to transportation costs or η for shocks to market size) at territory j as:

$$\mathbb{E}[v_i|j,\sigma].$$

The true effect on expected observed violence at i of a shock of size σ at j is

$$\delta_i(j,\sigma) = \mathbb{E}[v_i|j,\sigma] - \mathbb{E}[v_i|j,1].$$

Difference-in-differences estimates the change in expected observed violence at i vs. the

[16]See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Berman, Shapiro and Felter (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (Forthcoming); Maystadt and Ecker (2014); Mitra and Ray (2014).

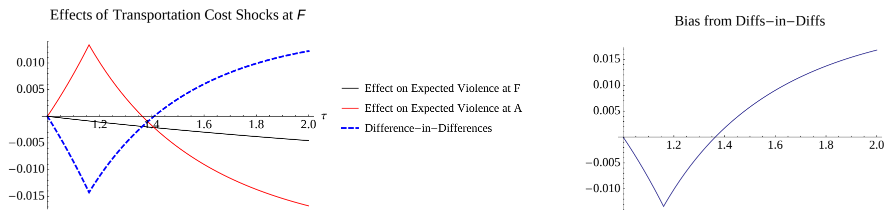

Figure 6.9: Difference-in-differences for the effect of a local transportation cost shock.

change in expected observed violence at k following a shock of size σ at i:

$$\Delta_{i,k}(i, \sigma) = \delta_i(i, \sigma) - \delta_k(i, \sigma).$$

6.3.1 Transportation Cost Shocks

As we've seen, a shock to transportation costs at F decreases expected observed violence at F, but has a non-monotone effect at A. Because of this violation of parallel trends, difference-in-differences does not recover the true effect of the shock at F on expected observed violence at F. More disturbingly, the direction of the bias depends on the size of the shock. For small shocks (so the effect of τ at A is positive), difference-in-differences overestimates the magnitude of the effect (i.e., says it is more negative than it is). For sufficiently large shocks (so the effect of τ at A is sufficiently negative), difference-in-differences actually gets the sign of the effect wrong (i.e., says the effect is positive). For some moderate shocks (so the effect of τ at A is close to zero), difference-in-differences comes close to the true effect. Thus, the empirical researcher cannot know the sign of the bias, or even the sign of the true effect, from difference-in-differences using A as the baseline.

These facts are illustrated in Figure 6.9. The left-hand panel shows the effect of a shock of size τ to transportation costs at F on expected observed violence at F ($\delta_F(F, \tau)$), on expected observed violence at A ($\delta_A(F, \tau)$), and on the difference-in-differences ($\Delta_{F,A}(F, \tau)$). The right-hand side shows that the sign of the bias ($\Delta(\tau) - \delta_F(\tau)$) can be positive or negative, depending on the size of the shock. When the bias is positive and larger in magnitude than the true effect, difference-in-differences gets the sign of the effect wrong.

6.3.2 Market Size

As we've already seen, a shock to market size at F increases expected observed violence at F and decreases expected observed violence at A. Again, because of the violation of

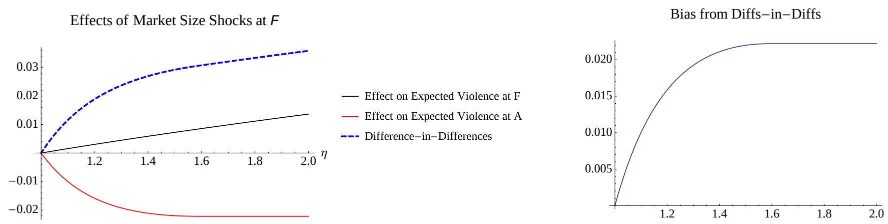

Figure 6.10: Difference-in-differences for market size shocks.

parallel trends, difference-in-differences does not recover the true effect of a shock at F on expected observed violence at F. But, unlike the case of transportation cost shocks, here we know the sign of the bias—difference-in-differences overestimates the effect.

This fact is illustrated in Figure 6.10. The left-hand panel shows the effect of a shock to market size at F of size η on expected observed violence at F ($\delta_F(F, \eta)$), on expected observed violence at A ($\delta_A(F, \eta)$), and on the difference-in-differences ($\Delta_{F,A}(F, \eta)$). The right-hand side shows that the bias associated with the difference-in-differences ($\Delta_{F,A}(F, \eta) - \delta_F(F, \eta)$) is positive and its magnitude is increasing in the size of the shock.

6.3.3 More Distant Controls

A standard empirical practice is to exclude neighboring territories when there is concern about spillovers. The model allows us to probe the efficacy of this approach. In particular, rather than using territory A as the baseline for a difference-in-differences estimate of the effect of a shock at F on observable violence at F, we can use territories C or D.

In this section, I present only the results on bias. Appendix F characterizes equilibrium observable violence at C and D in response to each type of shock at F.

Let's start with the case of transportation costs. The left-hand panel of Figure 6.11 shows that expected observed violence at C and D is monotonically increasing in transportation costs at F. As transportation costs at F increase, prices everywhere increase, making new territory more valuable and increasing conflict. This is an important difference between the more distant territories (C or D) and the nearest neighbor (A), where violence is non-monotone in transportation costs at F.

The right-hand panel of Figure 6.11 compares the bias from difference-in-differences using A, C, or D as the baselines. That figure shows two facts. First, using more distant territories as the baseline is not guaranteed to reduce bias (though it does so for some values of the shock). Second, since shocks at F increase violence at C and D, difference-

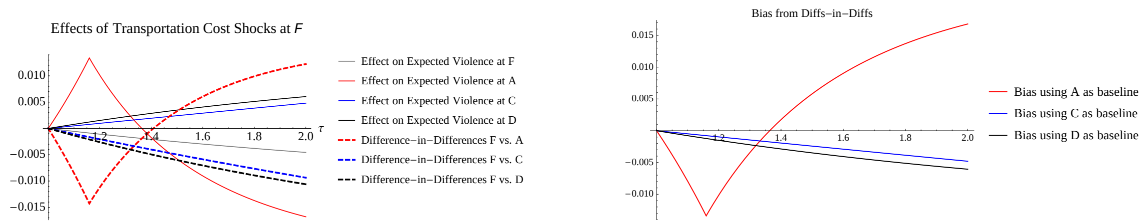

Figure 6.11: Difference-in-differences with more distant controls for transportation cost shocks.

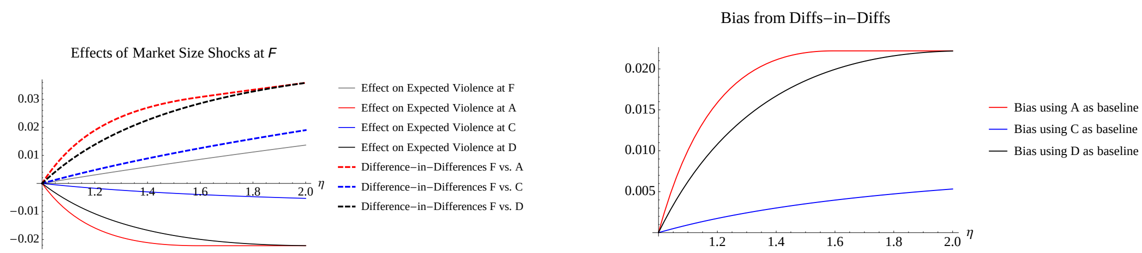

Figure 6.12: Difference-in-differences with more distant controls for market size shocks.

in-differences using C or D as the baseline always gives an overestimate of the magnitude of the true effect (i.e., produces an estimate that is too negative). Thus, the difference-in-differences estimate using distant territories should be regarded as an upper bound on the magnitude of the effect of a transportation cost shock.

Next consider the case of market size. The left-hand panel of Figure 6.12 shows that expected observed violence at C and D is decreasing in market size at F. As market size at F increases, prices decrease, making new territory less valuable. The effect on conflict is in the same direction as it is at A. Because the more distant territories are more insulated from the shock, the magnitude is smaller.

The right-hand panel of Figure 6.12 compares the bias from difference-in-differences using A, C, or D as the baselines. That figure shows two facts. First, using more distant territories as the baseline reduces bias. Second, since shocks at F decrease violence at C and D, difference-in-differences using C or D still gives an overestimate of the magnitude of the true effect (i.e., produces an estimate that is too positive). Thus, the difference-in-differences estimate using distant territories is less biased than using nearby territories, confirming standard practice. But it is still an upper bound on the magnitude of the effect of a market size shock.

7 Conclusion

I study a model of armed factions fighting over control of territories from which they endogenously extract economic rents. The analysis, building on canonical models of both conflict and spatial price competition, yields several results worth reemphasizing.

First, local and global changes to market conditions have different effects on conflict. Most of the modern empirical literature exploits local variation. Yet, the model's predictions about the effects of local changes are different from conventional hypotheses (which are more similar to the model's predictions regarding global changes). In particular, the model predicts that changes to local economic that increase factions' overall rents are associated with a decrease in expected observed violence.

Second, local economic shocks affect conflict at other territories. In the presence of such spillovers, a difference-in-differences research design, of course, produces biased estimates. The model allows us to explore the sign and magnitude of the bias theoretically. In the case of shocks to local market size, the bias is always positive, so that difference-in-differences leads to overestimates. Using territories more distant from the shock as the baseline for comparison reduces, but does not eliminate, the bias. In the case of shocks to local transportation costs (market power), when comparing to a territory neighboring the shocked territory, the sign and magnitude of the bias depend on the magnitude of the shock and, thus, the empirical researcher can learn neither the magnitude nor the sign of the effect. Using territories more distant from the shock as the baseline for comparison may increase or decrease bias, but does guarantee the direction of bias. Unfortunately, again, the bias is such that difference-in-differences overestimates the effect size.

Both the divergence between the local and the global comparative statics, and the usefulness of the local comparative statics for understanding what difference-in-differences estimates in such a setting, highlight a complementarity between identification-oriented, micro-empirical scholarship on conflict and theoretical models within which we can think about the sources of variation used in such studies.

Third, qualitative accounts and conventional wisdom suggest that an increase in the number of armed factions leads to an increase in observed violence. Here, the predicted relationship is more nuanced. An increase in factionalization increases the frequency of observed violence. However, when violence occurs, the more factions, the less intense it is. Highly factionalized environments, then, are characterized by frequent, low-level conflict and instability of the pattern of territorial control. Consolidated environments are characterized by infrequent, high-level conflict and stability of the pattern of territorial control. The overall expected amount of observed violence is non-monotone in the number of factions.

Finally, the model highlights a conceptual point. The results here arise because conflict outcomes feedback into economic behavior, which affects the returns to winning the conflict. Hence, the model demonstrates the importance of a political economy approach to the study of conflict that takes seriously the two-way relationship beween economic consequences and conflict outcomes.

References

Angrist, Joshua D. and Adriana D. Kugler. 2008. "Rural Windfall or a New Resource Curse? Coca, Income, and Civil Conflict in Colombia." *Review of Economics and Statistics* 90(2):191–215.

Baye, Michael R., Dan Kovenock and Casper G. De Vries. 1996. "The All-Pay Auction with Complete Information." *Economic Theory* 8(2):291–305.

Bazzi, Samuel and Christopher Blattman. 2014. "Economic Shocks and Conflict: The Evidence from Commodity Prices." *American Economic Journal: Macroeconomics* 6(4):1–38.

Beittel, June S. 2013. "Mexico's Drug Trafficking Organizations: Source and Scope of the Violence." Congressional Research Service Report for Congress 7-5700.

Berman, Eli, Jacob N. Shapiro and Joseph H. Felter. 2011. "Can Hearts and Minds be Bought? The Economics of Counterinsurgency in Iraq." *Journal of Political Economy* 119:766–819.

Besley, Timothy and Hannes Mueller. 2012. "Estimating the Peace Dividend: The Impact of Violence on House Prices in Northern Ireland." *American Economic Review* 102(2):810–833.

Besley, Timothy and Torsten Persson. 2010. "State Capacity, Conflict, and Development." *Econometrica* 78(1):1–34.

Besley, Timothy and Torsten Persson. 2011. "The Logic of Political Violence." *Quarterly Journal of Economics* 126(3):1411–1445.

Bremer, Stuart A. 1992. "Dangerous Dyads: Conditions Affecting the Likelihood of Interstate War, 1816–1965." *Journal of Conflict Resolution* 36:309–?341.

Brückner, Markus and Antonio Ciccone. 2010. "International Commodity Prices, Growth and the Outbreak of Civil War in Sub-Saharan Africa." *The Economic Journal* 120(544):519–534.

Carter, David B. 2010. "The Strategy of Territorial Conflict." *American Journal of Political Science* 54(4):969–987.

Caselli, Francesco, Massimo Morelli and Dominic Rohner. 2015. "The Geography of Interstate Resource Wars." *Quarterly Journal of Economics* 130(1):267–315.

Castillo, Juan Damilo, Daniel Mejia and Pascual Restrepo. 2013. "Illegal Drug Markets and Violence in Mexico: The Causes Beyond Calderon." Universidad de los Andes typescript.

Chassang, Sylvain and Gerard Padro i Miguel. 2009. "Economic Shocks and Civil War." *Quarterly Journal of Political Science* 4(3):211–228.

Crost, Benjamin, Joseph Felter and Patrick Johnston. 2014. "Aid Under Fire: Development Projects and Civil Conflict." *American Economic Review* 104(6):1833–1856.

Dal Bó, Ernesto and Pedro Dal Bó. 2011. "Workers, Warriors, and Criminals: Social Conflict in General Equilibrium." *Journal of the European Economic Association* 9(4):646–677.

Deininger, Klaus. 2003. "Causes and Consequences of Civil Strife: Micro-Level Evidence from Uganda." *Oxford Economic Papers* 55(4):579–606.

Dell, Melissa. Forthcoming. "Trafficking Networks and the Mexican Drug War." *American Economic Review* .

Dube, Oeindrila and Juan Vargas. 2013. "Commonity Price Shocks and Civil Conflict: Evidence from Colombia." *Review of Economic Studies* 80(4):1384–1421.

Dube, Oeindrila, Omar García-Ponce and Kevin Thom. Forthcoming. "From Maize to Haze: Trade Liberalization and the Growth of the Mexican Drug Sector." *Journal of the European Economic Association* .

Epstein, Gil S. and Ira N. Gang. 2007. "Who is the Enemy?" *Defence and Peace Economics* 18(6):469–484.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fearon, James D. 2008*a*. Economic Development, Insurgency, and Civil War. In *Institutions and Economic Performance*, ed. Elhanan Helpman. Cambridge, MA: Harvard University Press pp. 292–328.

Fearon, James D. 2008*b*. "A Simple Political Economy of Relations among Democracies and Autocracies." Stanford University typescript.

Grossman, Herschel I. 1999. "Kleptocracy and Revolutions." *Oxford Economic Papers* 51(2):267–283.

Hafer, Catherine. 2006. "On the Origins of Property Rights: Conflict and Production in the State of Nature." *Review of Economic Studies* 73(1):119–143.

Hidalgo, F. Daniel, Suresh Naidu, Simeon Nichter and Neal Richardson. 2010. "Economic Determinants of Land Invasions." *Review of Economics and Statistics* 92(3):505–523.

Hillman, Arye L. and John G. Riley. 1989. "Politically Contestable Rents and Transfers." *Economics & Politics* 1(1):17–39.

Hirshleifer, Jack. 1991. "The Technology of Conflict as an Economic Activity." *American Economic Review (Papers & Proceedings)* pp. 130–134.

Krishna, Vijay and John Morgan. 1997. "An Analysis of the War of Attrition and the All-Pay Auction." *Journal of Economic Theory* 72(2):343–362.

Lessing, Benjamin. 2013. When Business Gets Bloody: State Policy and Drug Violence. In *Small Arms Survey 2012*. New York: Cambridge University Press.

Maystadt, Jean-François and Olivier Ecker. 2014. "Extreme Weather and Civil War: Does Drought Fuel Conflict in Somalia through Livestock Price Shocks?" *American Journal of Agricultural Economics* 96:1157–1181.

Mejia, Daniel and Pascual Restrepo. 2013. "Bushes and Bullets: Illegal Cocaine Markets and Violence in Colombia." Documento CEDE No. 2013-53.

Mitra, Anirban and Debraj Ray. 2014. "Implications of an Economic Theory of Conflict: Hindu-Muslim Violence in India." *Journal of Political Economy* 122(4):719–765.

Papachristos, Andrew V., David M. Hureau and Anthony A. Braga. 2013. "The Corner and the Crew: The Influence of Geography and Social Networks on Gang Violence." *American Sociological Review* 78(3):417–447.

Powell, Robert. 2004. "The Inefficient Use of Power: Costly Conflict with Complete Information." *American Political Science Review* 98(2):231–241.

Reed, William and Daina Chiba. 2010. "Decomposing the Relationship Between Contiguity and Militarized Conflict." *American Journal of Political Science* 54:61–73.

Rios, Viridiana. 2013. "Why Did Mexico Become So Violent? A Self-Reinforcing Violent Equilibrium Caused by Competition and Enforcement." *Trends in Organized Crime* 16(2):138–155.

Rohner, Dominic, Mathias Thoenig and Fabrizio Zilibotti. 2013. "War Signals: A Theory of Trade, Trust, and Conflict." *Review of Economic Studies* 80(3):1114–1147.

Vasquez, John A. 1995. "Why Do Neighbors Fight? Proximity, Interaction, or Territoriality." *Journal of Peace Research* 32:277–293.

Appendices
"Factional Conflict and Territorial Rents"

A Factionalization

Proof of Lemma 4.1. First note that

$$u^{\mathbf{2},1,1,1,1} = \frac{145t}{2166} > \frac{t}{36} = u^{1,1,1,1,1,1}.$$

Hence, $\mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1}(\pi)$ is minimized at $\pi = 0$, where we have

$$\mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1}(0) = \frac{65t}{2166} > \frac{t}{36} = \mathrm{IR}_{\mathrm{def}}^{1,1,1,1,1,1}.$$

■

Proof of Proposition 4.4.

- First consider frequency. It follows from Proposition 4.1 that the probability of observing violence with six factions is 1.

  With three symmetric-connected factions, there are no safe territories. Thus, from Proposition 4.2, the probability of observing violence is

  $$\frac{\mathrm{IR}_{\mathrm{def}}^{2,2,2}}{\mathrm{IR}_{\mathrm{att}}^{2,2,2}} = \frac{112,288}{153,579} < 1.$$

  With two symmetric-connected factions, conditional on a border territory being vulnerable, the probability of observing violence is

  $$\frac{\mathrm{IR}_{\mathrm{def}}^{3,3}}{\mathrm{IR}_{\mathrm{att}}^{3,3}} = \frac{77}{103}.$$

  A border territory is vulnerable with probability 2/3. In the other 1/3 of cases, the probability of observing violence is zero. Hence, the overall probability of observing violence with two symmetric-connected factions is

  $$\frac{2}{3} \cdot \frac{77}{103} = \frac{154}{309} < \frac{112,288}{153,579}.$$

- Next consider intensity. In all symmetric-connected configurations, $\mathbb{E}[v|v > 0]$ is the incremental return of the faction that values winning the second most. From Propositions 4.1, 4.2, and 4.3, those incremental returns are:

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = \frac{65Nt}{2166} < \text{IR}_{\text{def}}^{2,2,2} = \frac{28,072Nt}{660,969} < \text{IR}_{\text{def}}^{3,3} = \frac{77Nt}{1296}.$$

- Next consider variance. A random variable whose distribution places mass α on zero and mass $1-\alpha$ on a symmetric triangular distribution on $[0, b]$ has variance:

$$\left[\int_0^{\frac{b}{2}} x^2 \cdot \frac{(1-\alpha)x}{b^2}\,dx + \int_{\frac{b}{2}}^{b} x^2 \cdot \frac{(1-\alpha)(b-x)}{b^2}\,dx\right] - \left[\int_0^{\frac{b}{2}} x \cdot \frac{(1-\alpha)x}{b^2}\,dx + \int_{\frac{b}{2}}^{b} x \cdot \frac{(1-\alpha)(b-x)}{b^2}\,dx\right]^2 = \frac{(1+5\alpha-6\alpha^2)b^2}{24}. \quad (9)$$

In the case of 6 factions, observed violence is such a random variable with $\alpha = 0$ and $b = 2\text{IR}_{\text{att}}^{1,1,1,1,1,1}(0)$. Plugging these into Equation 9 yields

$$\text{var}[v|1,1,1,1,1,1] = \frac{4225N^2t^2}{28,149,336} \approx 0.00015N^2t^2.$$

In the case of 3 symmetric-connected factions, observed violence is such a random variable with $\alpha = 1 - \frac{\text{IR}_{\text{def}}^{2,2,2}}{\text{IR}_{\text{att}}^{2,2,2}}$ and $b = 2\text{IR}_{\text{def}}^{2,2,2}$. Plugging these into Equation 9 yields

$$\text{var}[v|2,2,2] = \frac{5,918,682,193,315,302,400N^2t^2}{10,304,474,604,431,881,718,601} \approx 0.00057N^2t^2$$

In the case of 2 symmetric-connected factions, observed violence is such a random variable with $\alpha = \frac{1}{3} + \frac{2}{3}\left(1 - \frac{\text{IR}_{\text{def}}^{3,3}}{\text{IR}_{\text{att}}^{3,3}}\right)$ and $b = 2\text{IR}_{\text{def}}^{3,3}$. Plugging these into Equation 9 yields

$$\text{var}[v|3,3] = \frac{188,548,129N^2t^2}{160,371,415,296} \approx 0.00118N^2t^2.$$

- The results on overall expected observed violence follow from Table 4.2.

$\blacksquare$

B Global Comparative Statics

Proof of Proposition 5.1.

The results on expected observed violence follow directly from calculations reported in Table 4.1. The results on variance follow direction from calculations in the proof of Proposition 4.4.

The probability of transitioning from 6 factions to 5 factions is 1, which is constant in t and N. The probability of transitioning from $2,2,2$ to $3,2,1$ is

$$\frac{\mathrm{IR}^{2,2,2}_{\mathrm{def}}}{\mathrm{IR}^{2,2,2}_{\mathrm{att}}} \cdot \frac{1}{2} = \frac{\frac{28{,}072Nt}{660{,}969}}{\frac{51{,}193Nt}{881{,}292}} \cdot \frac{1}{2},$$

which is constant in N and t. The probability of transitioning from $3,3$ to $4,2$ is

$$\frac{2}{3} \cdot \frac{\mathrm{IR}^{3,3}_{\mathrm{def}}}{\mathrm{IR}^{3,3}_{\mathrm{att}}} \cdot \frac{1}{2} = \frac{2}{3} \cdot \frac{77}{103} \cdot \frac{1}{2},$$

which is constant in t and N. ■

C Local Comparative Statics

In this appendix I provide proofs of results for the local comparative statics. Characterization of the economic equilibria are in Appendix E.

C.1 Proofs for Local Market Size Shocks

Proof of Proposition 6.1. From Appendix E.1, rents under ABC, DEF and under $ABCF, DE$ are:

$$u^{\mathbf{ABC},DEF}(\eta) = \frac{(602 + 6408\eta + 23371\eta^2 + 32308\eta^3 + 11724\eta^4 + 512\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2} \tag{10}$$

$$u^{ABC,\mathbf{DEF}}(\eta) = \frac{(410 + 4752\eta + 19315\eta^2 + 31492\eta^3 + 16908\eta^4 + 2048\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}, \tag{11}$$

$$u^{\mathbf{ABCF},DE}(\eta) = \frac{(2408 + 26576\eta + 100262\eta^2 + 146966\eta^3 + 71201\eta^4 + 6728\eta^5)Nt}{324(4 + 24\eta + 29\eta^2)^2}$$

$$u^{ABCF,\mathbf{DE}}(\eta) = \frac{(820 + 9208\eta + 34069\eta^2 + 44527\eta^3 + 14503\eta^4 + 841\eta^5)Nt}{162(4 + 24\eta + 29\eta^2)^2}.$$

Hence, if conflict is over F, the incremental returns are:

$$\mathrm{IR}^{\text{pop}}_{\text{att},F}(\eta) = u^{\mathbf{ABCF},DE}(\eta) - u^{\mathbf{ABC},DEF}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 10238420\eta^8 + 22459372\eta^7 + 23035725\eta^6 + 13031928\eta^5 + 4314594\eta^4 + 833112\eta^3 + 86872\eta^2 + 3776\eta\right) Nt}{1296\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2}$$

and

$$\mathrm{IR}^{\text{pop}}_{\text{def},F}(\eta) = u^{ABC,\mathbf{DEF}}(\eta) - u^{ABCF,\mathbf{DE}}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 8999020\eta^8 + 17389508\eta^7 + 16381611\eta^6 + 8805816\eta^5 + 2828202\eta^4 + 535560\eta^3 + 55064\eta^2 + 2368\eta\right) Nt}{1296\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2}.$$

(i) Differentiating the rents, we have

$$\frac{\partial u^{\mathbf{ABC},DEF}(\eta)}{\partial \eta} = \frac{\left(2048\eta^6 + 4608\eta^5 - 57608\eta^4 - 66596\eta^3 - 28434\eta^2 - 5485\eta - 408\right) Nt}{648\left(8\eta^2+6\eta+1\right)^3}.$$

This is negative if the numerator is negative. To see that this is the case, notice that for any $\eta \in [1,2]$, $2048\eta^6 < 57608\eta^4$ and $4608\eta^5 < 66596\eta^3$, so the positive terms are more than off-set by the negative terms.

$$\frac{\partial u^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta} = \frac{\left(8192\eta^6 + 18432\eta^5 - 19400\eta^4 - 26228\eta^3 - 9786\eta^2 - 1501\eta - 84\right) Nt}{648\left(8\eta^2+6\eta+1\right)^3}$$

To see that rents are decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{Nt}{72} < 0$ and at $\eta = 2$ it is $\frac{91943Nt}{9841500} > 0$. To see that rents are convex, differentiate again:

$$\frac{\partial^2 u^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta^2} = \frac{\left(580736\eta^5 + 605232\eta^4 + 235552\eta^3 + 40072\eta^2 + 2472\eta + 11\right) Nt}{648\left(8\eta^2+6\eta+1\right)^4},$$

which is clearly positive for $\eta \in [1,2]$.

$$\frac{\partial u^{\mathbf{ABCF},DE}(\eta)}{\partial \eta} = \frac{\left(97556\eta^6 + 242208\eta^5 - 354903\eta^4 - 574398\eta^3 - 274260\eta^2 - 57528\eta - 4640\right) Nt}{162\left(29\eta^2+24\eta+4\right)^3}$$

To see that rents decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{5Nt}{162} < 0$ and at $\eta = 2$ it is $\frac{23Nt}{7056} > 0$. To see that it rents are convex, differentiate again:

$$\frac{\partial^2 u^{\mathbf{ABCF},DE}(\eta)}{\partial \eta^2} = \frac{\left(1919539\eta^5 + 2572173\eta^4 + 1451984\eta^3 + 446168\eta^2 + 76368\eta + 5776\right) Nt}{9\left(29\eta^2 + 24\eta + 4\right)^4},$$

which is clearly positive for $\eta \in [1,2]$.

$$\frac{\partial u^{ABCF,\mathbf{DE}}(\eta)}{\partial \eta} = \frac{\left(24389\eta^6 + 60552\eta^5 - 578319\eta^4 - 675306\eta^3 - 266772\eta^2 - 43560\eta - 2528\right) Nt}{162\left(29\eta^2 + 24\eta + 4\right)^3}$$

To see that this is negative, notice that for any $\eta \in [1,2]$, $24389\eta^6 < 578319\eta^4$ and $60552\eta^5 < 675306\eta^3$, so the positive terms are more than off-set by the negative terms.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)}{\partial \eta} = \frac{Nt}{648\left(8\eta^2 + 6\eta + 1\right)^3 \left(29\eta^2 + 24\eta + 4\right)^3}$$
$$\Big[149846016\eta^{12} + 709185024\eta^{11} + 1804009768\eta^{10} + 3180547444\eta^9 + 3868730394\eta^8 + 3194448545\eta^7$$
$$+ 1799287064\eta^6 + 696886116\eta^5 + 185369396\eta^4 + 33247640\eta^3 + 3837552\eta^2 + 256864\eta + 7552\Big]$$

and

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta)}{\partial \eta} = \frac{Nt}{648\left(8\eta^2 + 6\eta + 1\right)^3 \left(29\eta^2 + 24\eta + 4\right)^3}$$
$$\Big[149846016\eta^{12} + 709185024\eta^{11} + 1938493592\eta^{10} + 3288362780\eta^9 + 3601960590\eta^8 + 2665205479\eta^7$$
$$+ 1372937176\eta^6 + 498251100\eta^5 + 126687292\eta^4 + 22024216\eta^3 + 2485200\eta^2 + 163424\eta + 4736\Big],$$

both of which are clearly positive for any $\eta \in [1,2]$.

(iii) In the event that an interior territory is vulnerable, observed violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's.

Subtracting, we have:

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)-\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta)=\frac{\left(619700\eta^8+2534932\eta^7+3327057\eta^6+2113056\eta^5+743196\eta^4+148776\eta^3+15904\eta^2+704\eta\right)Nt}{648\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2},$$

which is clearly positive for any $\eta\in[1,2]$.

Thus, expected observed violence is

$$\frac{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)}=\frac{\left(1291776\eta^9+8999020\eta^8+17389508\eta^7+16381611\eta^6+8805816\eta^5+2828202\eta^4+535560\eta^3+55064\eta^2+2368\eta\right)^2Nt}{1296(8\eta^2+6\eta+1)^2(29\eta^2+24\eta+4)^2(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\eta}\frac{\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def},F}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)}=\frac{\left(1291776\eta^8+8999020\eta^7+17389508\eta^6+16381611\eta^5+8805816\eta^4+2828202\eta^3+535560\eta^2+55064\eta+2368\right)Nt}{648(8\eta^2+6\eta+1)^3(29\eta^2+24\eta+4)^3(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)^2}$$
$$\times\Big[193567487164416\eta^{20}+2636013792927744\eta^{19}+14942866864822272\eta^{18}+51819230507149024\eta^{17}+122367280695000336\eta^{16}+206967166643804864\eta^{15}+259890474116763824\eta^{14}+249193190122341378\eta^{13}+186403803800274835\eta^{12}+110473424142844948\eta^{11}+52399600963659870\eta^{10}+19995986823887684\eta^9+6143372086092296\eta^8+1513744764870168\eta^7+296519405242384\eta^6+45491642345344\eta^5+5339965611648\eta^4+462318253184\eta^3+27777575168\eta^2+1032932352\eta+17883136\Big],$$

which is clearly positive for any $\eta\in[1,2]$.

■

Proof of Proposition 6.2. From Appendix E.1, rents under $BC, ADEF$ are:

$$u^{\mathbf{BC},ADEF}(\eta)=\frac{\left(313\eta^4+4686\eta^3+20497\eta^2+20532\eta+5956\right)Nt}{81(35\eta+22)^2}$$

$$u^{BC,\mathbf{ADEF}}(\eta)=\frac{\left(11744\eta^4+103041\eta^3+278285\eta^2+246312\eta+68900\right)Nt}{648(35\eta+22)^2},$$

and rents under ABC, DEF are reported in Equations 10 and 11.

Hence, if conflict is over A, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},A}(\eta)&=u^{BC,\mathbf{ADEF}}(\eta)-u^{ABC,\mathbf{DEF}}(\eta)\\&=\frac{\left(1503232\eta^8+12935296\eta^7+32759508\eta^6+30349524\eta^5+7207189\eta^4-5078258\eta^3-3535132\eta^2-785144\eta-60640\right)Nt}{1296(35\eta+22)^2(8\eta^2+6\eta+1)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},A}(\eta) &= u^{\mathbf{ABC},DEF}(\eta) - u^{\mathbf{BC},ADEF}(\eta)\\ &= -\frac{\left(320512\eta^8+4652032\eta^7+13296660\eta^6-1413060\eta^5-28463891\eta^4-29236772\eta^3-12691846\eta^2-2556488\eta-196072\right)Nt}{1296(35\eta+22)^2\left(8\eta^2+6\eta+1\right)^2}.\end{aligned}$$

(i) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},A}(\eta)}{\partial \eta} = \frac{Nt}{648(35\eta+22)^3\left(8\eta^2+6\eta+1\right)^3}$$
$$\Big[420904960\eta^{10}+2971436544\eta^9+8242655616\eta^8+13929007224\eta^7+15777138676\eta^6+11897551650\eta^5$$
$$+5832602097\eta^4+1807243525\eta^3+337953462\eta^2+34600700\eta+1490296\Big],$$

which is clearly positive for any $\eta \in [1, 2]$, and

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},A}(\eta)}{\partial \eta} = \frac{-Nt}{648(35\eta+22)^3\left(8\eta^2+6\eta+1\right)^3}$$
$$\Big[89743360\eta^{10}+898719744\eta^9+2854103040\eta^8+7300767576\eta^7+12496459804\eta^6+12803052270\eta^5$$
$$+7918942659\eta^4+2994912490\eta^3+680093556\eta^2+85613720\eta+4622656\Big],$$

which is clearly negative for any $\eta \in [1, 2]$.

(ii) Point (i) of this proposition shows the defender's incremental return is strictly decreasing. Thus, to show that an $\hat{\eta} \in (1, 2)$ exists, it suffices to show that the defender's incremental return is positive at $\eta = 1$ and negative at $\eta = 2$. At $\eta = 1$, the defender's incremental return is $\frac{77Nt}{1296} > 0$. At $\eta = 2$ the defender's incremental return is $-\frac{7686319Nt}{231384600} < 0$.

(iii) Given the previous results in this proposition, it now suffices to show that the defender's incremental return is less than the attacker's. Subtracting, this is the case if:

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att,\ A}}(\eta) - \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def,\ A}}(\eta) = \frac{\left(911872\eta^8+8793664\eta^7+23028084\eta^6+14468232\eta^5-10628351\eta^4-17157515\eta^3-8113489\eta^2-1670816\eta-128356\right)Nt}{648(35\eta+22)^2\left(8\eta^2+6\eta+1\right)^2} > 0.$$

Since the defender's incremental return is decreasing in η and the attacker's is increasing in η, the left-hand side is minimized at $\eta = 1$. Thus, it suffices to show that the inequality holds at $\eta = 1$. At $\eta = 1$, the inequality reduces to $\frac{13t}{648} > 0$.

■

C.2 Proofs of for Local Transportation Cost Shocks

Proof of Proposition 6.3. From Appendix E.2, the rents under ABC, DEF and under $ABCF, DE$ are:

$$u^{\mathbf{ABC},DEF}(\tau) = \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2} \tag{12}$$

$$u^{ABC,\mathbf{DEF}}(\tau) = \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2}, \tag{13}$$

$$u^{\mathbf{ABCF},DE}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right) Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2}$$

$$u^{ABCF,\mathbf{DE}}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right) Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

Hence, when a border territory is vulnerable, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, F}}(\tau) &= u^{\mathbf{ABCF},DE}(\tau) - u^{\mathbf{ABC},DEF}(\tau) \\ &= \frac{\left(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154\right)Nt}{648(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{def, F}}(\tau) &= u^{ABC,\mathbf{DEF}}(\tau) - u^{ABCF,\mathbf{DE}}(\tau) \\ &= \frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)Nt}{2592(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}.\end{aligned}$$

(i) Differentiating the rents, we have

$$\frac{\partial u^{\mathbf{ABC},DEF}(\tau)}{\partial \tau} = \frac{\left(6360\tau^5 + 108628\tau^4 + 663482\tau^3 + 1800891\tau^2 + 2005820\tau + 760819\right) Nt}{324(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial u^{ABC,\mathbf{DEF}}(\tau)}{\partial \tau} = \frac{\left(8664\tau^5 + 203140\tau^4 + 1567538\tau^3 + 5015871\tau^2 + 5991404\tau + 2279383\right) Nt}{1296(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial u^{\mathbf{ABCF},DE}(\tau)}{\partial \tau} = \frac{\left(61468\tau^5 + 1026583\tau^4 + 6237580\tau^3 + 16551342\tau^2 + 17968716\tau + 6551415\right)Nt}{648(\tau+1)^2(\tau+5)^2(4\tau+15)^3}$$

$$\frac{\partial u^{ABCF,\mathbf{DE}}(\tau)}{\partial \tau} = \frac{\left(13090\tau^5 + 212431\tau^4 + 1270000\tau^3 + 3351690\tau^2 + 3660714\tau + 1369035\right)Nt}{162(\tau+1)^2(\tau+5)^2(4\tau+15)^3},$$

all of which are clearly positive for any $\tau \in [1,2]$.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,F}}(\tau)}{\partial \tau} = \frac{-Nt}{648(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$
$$\Big[322336\tau^{10}+10451448\tau^9+143061988\tau^8+1073013626\tau^7+4766618725\tau^6+12523786196\tau^5$$
$$+ 17710031949\tau^4 + 8367954734\tau^3 - 7878111669\tau^2 - 8896414788\tau - 1152922545\Big]$$

and

$$\frac{\partial \mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,F}}(\tau)}{\partial \tau} = \frac{-Nt}{1296(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$
$$\Big[283264\tau^{10}+10174464\tau^9+163482400\tau^8+1529546792\tau^7+9107162500\tau^6+35555048270\tau^5$$
$$+90894354783\tau^4+148662284540\tau^3+149453302806\tau^2+87077604366\tau+24236491815\Big].$$

The incremental returns are decreasing if the arguments in square brackets are positive. This is clearly the case for the defender for any $\tau \in [1,2]$. Now consider the attacker. To see that the term in the square brackets is positive in this case, note that for any $\tau \in [1,2]$, $8367954734\tau^3 > 1152922545$, $17710031949\tau^4 > 8896414788\tau$, and $12523786196\tau^5 > 7878111669\tau^2$, so each negative terms is more than off-set by a separate positive term.

(iii) In the event that an interior territory is vulnerable, observed violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,\,F}}(\tau) - \mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,\,F}}(\tau) = \frac{\left(90352\tau^5 + 1713756\tau^4 + 11545728\tau^3 + 33407975\tau^2 + 39254262\tau + 15356727\right)Nt}{2592(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2} > 0,$$

which holds for any $\tau \in [1,2]$.

Thus, expected observed violence is

$$\frac{\mathrm{IR}^{\text{trans}}_{\text{def, F}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, F}}(\tau)} = \frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)^2 Nt}{10368(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\tau}\frac{\mathrm{IR}^{\text{trans}}_{\text{def, F}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, F}}(\tau)} = \frac{-\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)Nt}{10368(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)^2}$$

$$\times \Big[22826546176\tau^{16}+1346834559616\tau^{15}+37276519674400\tau^{14}+639371782994576\tau^{13}+ 7567435768222208\tau^{12}+65199087795895376\tau^{11}+420975308247002594\tau^{10}+2069002610638570577\tau^{9}+ 7793137617277828811\tau^8 + 22498728719469456958\tau^7 + 49489440661438539010\tau^6 + 81914683489662021400\tau^5 + 99928825843407467628\tau^4 + 86905973146295199618\tau^3 + 50946917644932029964\tau^2 + 18077056655295975543\tau + 2945458294230415545\Big]$$

which is clearly negative for any $\tau \in [1,2]$.

■

Proof of Proposition 6.4.

From Appendix E.2, rents under $BC, ADEF$ are:

$$u^{BC,\mathbf{ADEF}}(\tau) = \frac{19\left(409\tau^2+3406\tau+7129\right)Nt}{324(11\tau+46)^2}$$

$$u^{\mathbf{BC},ADEF}(\tau) = \frac{\left(109193\tau^3+995320\tau^2+2701885\tau+1859858\right)Nt}{2592(\tau+1)(11\tau+46)^2},$$

and rents under ABC, DEF are reported in Equations 12 and 13.

Hence, when the neighboring territory is vulnerable, incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, A}}(\tau) &= u^{BC,\mathbf{ADEF}}(\tau) - u^{ABC,\mathbf{DEF}}(\tau)\\ &= -\frac{\left(161936\tau^6+3167436\tau^5+19358912\tau^4+19156131\tau^3-201237120\tau^2-623205917\tau-422130578\right)Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}\end{aligned}$$

and

$$\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau) = u^{\mathbf{ABC},DEF}(\tau) - u^{\mathbf{BC},ADEF}(\tau)$$
$$= \frac{\left(299164\tau^6+6053244\tau^5+45925507\tau^4+158823576\tau^3+229336425\tau^2+59685980\tau-49812496\right)Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}.$$

Note that the attacker's incremental return is positive because $422130578 > 161936\tau^6 + 3167436\tau^5 + 19358912\tau^4 + 19156131\tau^3$ for any $\tau \in [1,2]$.

(i) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)}{\partial \tau} = \frac{-Nt}{2592(\tau+1)^2(\tau+3)^2(2\tau+13)^3(11\tau+46)^3}$$
$$\Big[20676696\tau^8+757941316\tau^7+11271735818\tau^6+89934566643\tau^5+421079277994\tau^4+1174243638776\tau^3 + 1871978337534\tau^2 + 1524040258765\tau + 486909042458\Big],$$

which is clearly negative for any $\tau \in [1,2]$, and

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau)}{\partial \tau} = \frac{Nt}{324(\tau+1)^2(\tau+3)^2(2\tau+13)^3(11\tau+46)^3}$$
$$\Big[8440536\tau^8+250130612\tau^7+3134243962\tau^6+21644759691\tau^5+89552437616\tau^4+224745641977\tau^3 + 328095694158\tau^2 + 248494489970\tau + 74115939478\Big],$$

which is clearly positive for any $\tau \in [1,2]$.

(ii) Point (i) of this proposition implies that both incremental returns are monotone in τ and that observed violence is increasing in τ if $\mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau) < \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)$ and decreasing in τ if $\mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau) > \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)$. Hence, to show that a $\hat{\tau} \in (1,2)$ exists, it suffices to show that

$$\mathrm{IR}^{\text{trans}}_{\text{att},A}(1) - \mathrm{IR}^{\text{trans}}_{\text{def},A}(1) > 0 \qquad \text{and} \qquad \mathrm{IR}^{\text{trans}}_{\text{att},A}(2) - \mathrm{IR}^{\text{trans}}_{\text{def},A}(2) < 0.$$

Subtracting, we have:

$$\mathrm{IR}^{\mathrm{trans}}_{\mathrm{att},A}(\tau) - \mathrm{IR}^{\mathrm{trans}}_{\mathrm{def},A}(\tau) = \frac{-\left(760264\tau^6 + 15273924\tau^5 + 111209926\tau^4 + 336803283\tau^3 + 257435730\tau^2 - 503833957\tau - 521755570\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}.$$

At $\tau = 1$ this reduces to $\frac{13Nt}{648} > 0$ and at $\tau = 2$ it reduces to $-\frac{780541Nt}{8989056} < 0$, as required.

■

D Economic Equilibrium

D.1 Six Factions: $1, 1, 1, 1, 1, 1$

Suppose there are six factions, each of which controls one territory. If demand is characterized by Equation 4 at some vector of prices, profits from territory i are:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

Given the symmetry of the factions, equilibrium prices are characterized by the following condition:

$$N\left[\frac{2p^* - 2p^*}{2t} + \frac{1}{6}\right] - \frac{Np^*}{t} = 0.$$

This implies that in equilibrium the common price is

$$p^*_{1,1,1,1,1,1} = \frac{t}{6}$$

Note that for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ for all i, j, so demand is in fact characterized by Equation 4. Each faction's equilibrium rents are

$$u^{1,1,1,1,1,1} = \frac{t}{6} \cdot \frac{N}{6} = \frac{Nt}{36}.$$

D.2 Five Factions: $2, 1, 1, 1, 1$

Suppose there are five factions—one controlling two contiguous territories and all the remaining factions controlling one. Without loss of generality, suppose the large faction controls territories A and B. The there are three kinds of factions to consider:

(i) Large faction (controls A and B)

(ii) Border faction (controls C or F)

(iii) Interior faction (controls D or E)

If demand is characterized by Equation 4 at some vector of prices, the large faction's profits are:

$$N\left[p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)\right],$$

the C-border faction's profits are (the F-border faction is symmetric):

$$Np_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right),$$

and the $D-$interior faction's profits are (the E-interior faction is symmetric):

$$Np_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right).$$

An equilibrium is described by the following first-order and symmetry conditions:

$$\begin{aligned}
\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}&=0\\
\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}&=0\\
\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}&=0\\
p_A^*&=p_B^*\\
p_C^*&=p_F^*\\
p_D^*&=p_E^*.
\end{aligned}$$

This implies that in equilibrium we have:

$$p_A^*=p_B^*=\frac{5t}{19}\qquad p_C^*=p_F^*=\frac{11t}{57}\qquad p_D^*=p_E^*=\frac{10t}{57}.$$

Note that for any $t\leq 1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 4.

Rents for the large faction, the border factions, and the interior factions, respectively, are

$$u^{\mathbf{2},1,1,1,1} = \frac{145Nt}{2166} \qquad u^{2,\mathbf{1},1,1,\mathbf{1}} = \frac{40Nt}{1083} \qquad u^{2,1,\mathbf{1},\mathbf{1},1} = \frac{100Nt}{3249}.$$

D.3 Three Symmetric-Connected Factions: $2, 2, 2$

Suppose there are three factions, each controlling two contiguous territories. If demand is characterized by Equation 4 at some vector of prices, then a faction controlling territories i and $i+1$ has profits:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1}) \right] + p_{i+1} \left[D_{i+1}(p_{i+1}, p_{i+2}) + D_{i+1}(p_{i+1}, p_i) \right].$$

Given the symmetry, equilibrium prices are described by the following condition:

$$\frac{1}{6} - \frac{p^*}{t} + \frac{p^*}{2t} = 0,$$

which implies the following common price:

$$p^*_{2,2,2} = \frac{t}{3}$$

Notice $p^*_{2,2,2} > 2 - p^*_{2,2,2} - \frac{t}{6}$ for any $t \leq 1$, so demand is in fact characterized by Equation 4.

Equilibrium profits are:

$$u^{2,2,2} = \frac{t}{3} \cdot \frac{N}{3} = \frac{Nt}{9}.$$

D.4 Three Asymmetric Factions: $3, 2, 1$

Suppose there are three factions, one controlling three contiguous territories, one controlling two contiguous territories, and one controlling one territory. Without loss of generality, suppose the three factions are ABC, DE, F.

If demand is characterized by Equation 4 at some vector of prices, then the large faction's payoffs are

$$N \left[p_A \left(\frac{1}{6} + \frac{p_B + p_F - 2p_A}{2t} \right) + p_B \left(\frac{1}{6} + \frac{p_A + p_C - 2p_B}{2t} \right) + p_C \left(\frac{1}{6} + \frac{p_B + p_D - 2p_C}{2t} \right) \right],$$

the medium faction's payoffs are

$$N \left[p_D \left(\frac{1}{6} + \frac{p_C + p_E - 2p_D}{2t} \right) + p_E \left(\frac{1}{6} + \frac{p_D + p_F - 2p_E}{2t} \right) \right],$$

and the small faction's payoffs are

$$Np_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right).$$

Prices satisfy the following six first-order conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}+\frac{p_E^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_D^*+p_F^*-2p_E^*}{2t}-\frac{p_E^*}{t}+\frac{p_D^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_A^*+p_E^*-2p_F^*}{2t}-\frac{p_F^*}{t}=0$$

Solving, this implies the following equilibrium prices:

$$p_A^*=\frac{637t}{1626}\quad p_B^*=\frac{395t}{813}\quad p_C^*=\frac{112t}{271}$$

$$p_D^*=\frac{283t}{813}\quad p_E^*=\frac{175t}{542}$$

$$p_F^*=\frac{71t}{271}.$$

Note that for any $t\leq 1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 4.

These prices imply the following equilibrium rents:

$$u^{\mathbf{3},2,1}=\frac{447,343Nt}{2,643,878}\qquad u^{3,\mathbf{2},1}=\frac{298,831Nt}{2,643,876}\qquad u^{3,2,\mathbf{1}}=\frac{5041Nt}{73,441}.$$

D.5 Two Symmetric-Connected Factions: $3,3$

Suppose there are two factions, each controlling three contiguous territories. Without loss of generality, suppose the factions control A,B,C and D,E,F, respectively.

If demand is characterized by Equation 4 at some vector of prices, then a faction controlling territories $i-1$, i and $i+1$ has profits:

$$p_{i-1}\left[D_{i-1}(p_{i-1},p_i)+D_{i-1}(p_{i-1},p_{i-2})\right]+p_i\left[D_i(p_i,p_{i+1})+D_i(p_i,p_{i-1})\right]+p_{i+1}\left[D_{i+1}(p_{i+1},p_{i+2})+D_{i+1}(p_{i+1},p_i)\right].$$

Equilibrium prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$p_A^*=p_C^*=p_D^*=p_F^*$$

$$p_B^*=p_E^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^*=p_C^*=p_D^*=p_F^*=\frac{t}{2}\qquad p_B^*=p_E^*=\frac{7t}{12}.$$

Note that for any $t\leq 1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact described by Equation 4.

Equilibrium profits for each faction are:

$$u^{3,3}=\frac{37Nt}{144}.$$

D.6 Two Asymmetric Factions: $4,2$

Suppose there are two factions, one controlling four contiguous territories and one controlling two contiguous territories. Without loss of generality, suppose the two factions control A,B,C,D and E,F.

If demand is characterized by Equation 4 at some vector of prices, then the large faction's payoffs are:

$$N\left(p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)+p_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right)+p_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right)\right),$$

and the small faction's payoffs are:

$$N\left(p_E\left(\frac{1}{6}+\frac{p_D+p_F-2p_E}{2t}\right)+p_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right)\right).$$

In equilibrium, prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6} + \frac{p_B^* + p_F^* - 2p_A^*}{2t} - \frac{p_A^*}{t} + \frac{p_B^*}{2t} = 0$$

$$\frac{1}{6} + \frac{p_C^* + p_A^* - 2p_B^*}{2t} - \frac{p_B^*}{t} + \frac{p_A^*}{2t} + \frac{p_C^*}{2t} = 0$$

$$\frac{1}{6} + \frac{p_A^* + p_E^* - 2p_F^*}{2t} - \frac{p_F^*}{t} + \frac{p_E^*}{2t} = 0$$

$$p_A^* = p_D^*$$

$$p_B^* = p_C^*$$

$$p_E^* = p_F^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^* = p_D^* = \frac{5t}{9} \qquad p_B^* = p_C^* = \frac{13t}{18} \qquad p_E^* = p_F^* = \frac{4t}{9}.$$

Note, for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ for all i, j, so demand is in fact described by Equation 4.

These prices imply the following rents:

$$u^{\mathbf{4},2} = \frac{109Nt}{324} \qquad u^{4,\mathbf{2}} = \frac{16Nt}{81}.$$

E Economic Equilibria for Local Comparative Statics

E.1 Local Market Size

Without loss of generality, suppose the two factions start controlling A, B, C and D, E, F. To find the incremental returns, I start by characterizing equilibrium in the five scenarios: ABC, DEF, $ABCF, DE$, $BC, ADEF$, $AB, CDEF$, and $ABCD, EF$.

E.1.1 ABC, DEF

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(16\eta^2 + 94\eta + 25\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_B = \frac{\left(86\eta^2 + 185\eta + 44\right) t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_C = \frac{\left(46\eta^2 + 73\eta + 16\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_D = \frac{\left(50\eta^2 + 71\eta + 14\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_E = \frac{\left(106\eta^2 + 175\eta + 34\right) t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_F = \frac{\left(32\eta^2 + 86\eta + 17\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}.$$

These prices are consistent with $p_A < p_F$ and $p_E \geq p_F$ for any $\eta \in [1, 2]$. Hence, this case is a candidate for an equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(11\eta^2 + 74\eta + 50\right) t}{9\left(11\eta^2 + 15\eta + 4\right)}$$

$$p_B = \frac{\left(113\eta^2 + 341\eta + 176\right) t}{36(\eta + 1)(11\eta + 4)}$$

$$p_C = \frac{\left(29\eta^2 + 74\eta + 32\right) t}{9(\eta + 1)(11\eta + 4)}$$

$$p_D = \frac{\left(53\eta^2 + 161\eta + 56\right) t}{18(\eta + 1)(11\eta + 4)}$$

$$p_E = \frac{(4\eta + 17)t}{18(\eta + 1)}$$

$$p_F = \frac{\left(44\eta^2 + 161\eta + 65\right) t}{18(\eta + 1)(11\eta + 4)}.$$

These prices are inconsistent with $p_E < p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(13\eta^2 + 97\eta + 25\right)t}{9(23\eta + 7)}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{\left(10\eta^2 + 94\eta + 31\right)t}{9(23\eta + 7)}$$

$$p_D = \frac{\left(34\eta^2 + 163\eta + 73\right)t}{18(23\eta + 7)}$$

$$p_E = \frac{\left(58\eta^2 + 163\eta + 94\right)t}{18(23\eta + 7)}$$

$$p_F = \frac{\left(58\eta^2 + 187\eta + 25\right)t}{18(23\eta + 7)}.$$

These prices are inconsistent with $p_E < p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(2\eta + 13)t}{30}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{(4\eta + 41)t}{90}$$

$$p_D = \frac{(13 + 2\eta)t}{30}$$

$$p_E = \frac{(4\eta + 17)t}{36}$$

$$p_F = \frac{(14\eta + 31)t}{90}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

We have only one candidate for an equilibrium (case (i)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this candidate profile of prices, the prices are ordered as follows:

$$p_E > p_B > p_D > p_C > p_F > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between buying from D and E. This person is located at

$$x^*_{DE} = \frac{18\eta^2 + 23\eta + 4}{192\eta^2 + 144\eta + 24}.$$

This person prefers to buy the good as long as:

$$1 - p_D - x^*_{DE} t \geq 0.$$

This is true if and only if:

$$\frac{-254\eta^2 t + 576\eta^2 - 353\eta t + 432\eta - 68t + 72}{72\,(8\eta^2 + 6\eta + 1)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if $322\eta^2 + 79\eta + 4 \geq 0$, which is the case for any $\eta \in [0, 1]$.

Equilibrium rents are:

$$u^{\mathbf{ABC},DEF}(\eta) = \frac{(602 + 6408\eta + 23371\eta^2 + 32308\eta^3 + 11724\eta^4 + 512\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}$$

and

$$u^{ABC,\mathbf{DEF}}(\eta) = \frac{(410 + 4752\eta + 19315\eta^2 + 31492\eta^3 + 16908\eta^4 + 2048\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}.$$

E.1.2 $ABCF, DE$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(128\eta^2 + 372\eta + 241\right)t}{18(46\eta + 11)}$$

$$p_B = \frac{\left(92\eta^2 + 441\eta + 208\right)t}{18(46\eta + 11)}$$

$$p_C = \frac{\left(28\eta^2 + 186\eta + 71\right)t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(20\eta^2 + 165\eta + 43\right)t}{9(46\eta + 11)}$$

$$p_E = \frac{2\left(13\eta^2 + 84\eta + 17\right)t}{9(46\eta + 11)}$$

$$p_F = \frac{\left(64\eta^2 + 204\eta + 17\right)t}{9(46\eta + 11)}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(88\eta^2 + 453\eta + 200\right)t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_B = \frac{\left(139\eta^2 + 426\eta + 176\right)t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_C = \frac{(31\eta + 64)t}{9(11\eta + 8)}$$

$$p_D = \frac{\left(43\eta^2 + 129\eta + 56\right)t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_E = \frac{2\left(11\eta^2 + 69\eta + 34\right)t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_F = \frac{(22\eta + 73)t}{9(11\eta + 8)}.$$

These prices are inconsistent with $p_A < p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(233\eta^2 + 408\eta + 100\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_B = \frac{\left(263\eta^2 + 390\eta + 88\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_C = \frac{\left(103\eta^2 + 150\eta + 32\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_D = \frac{2\left(31\eta^2 + 69\eta + 14\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_E = \frac{\left(29\eta^2 + 165\eta + 34\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_F = \frac{\left(58\eta^2 + 177\eta + 50\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}.$$

These prices are consistent with $p_E < p_F < p_A$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(44\eta + 203)t}{342}$$

$$p_B = \frac{(32\eta t + 215t)}{342}$$

$$p_C = \frac{5(2\eta + 17)t}{171}$$

$$p_D = \frac{4(2\eta + 17)t}{171}$$

$$p_E = \frac{(11\eta + 65)t}{171}$$

$$p_F = \frac{(28\eta + 67)t}{171}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1,2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (iii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_B > p_A > p_C > p_F > p_D > p_E.$$

Hence, the worst-off population member is the one who is just indifferent between A and B. This person's position is

$$x^*_{AB} = \frac{19\eta^2 + 30\eta + 8}{348\eta^2 + 288\eta + 48}.$$

Plugging this in, the person indifferent between A and B prefers to purchase the good if:

$$1 - p_B - x^*_{AB} t \geq 0$$

which is true if and only if:

$$\eta^2(1044 - 583t) + \eta(864 - 870t) - 200t + 144 \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$ the inequality holds if and only if $461\eta^2 - 6\eta - 56 \geq 0$, which is true for any $\eta \in [1,2]$.

The equilibrium rents are

$$u^{\mathbf{ABCF},DE}(\eta) = \frac{(2408 + 26576\eta + 100262\eta^2 + 146966\eta^3 + 71201\eta^4 + 6728\eta^5)Nt}{324(4 + 24\eta + 29\eta^2)^2}$$

$$u^{ABCF,\mathbf{DE}}(\eta) = \frac{(820 + 9208\eta + 34069\eta^2 + 44527\eta^3 + 14503\eta^4 + 841\eta^5)Nt}{162(4 + 24\eta + 29\eta^2)^2}.$$

E.1.3 $BC, ADEF$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(64\eta^2 + 186\eta + 35\right) t}{9(46\eta + 11)}$$

$$p_B = \frac{\left(26\eta^2 + 165\eta + 37\right) t}{9(46\eta + 11)}$$

$$p_C = \frac{4\left(5\eta^2 + 42\eta + 10\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(28\eta^2 + 204\eta + 53\right) t}{9(46\eta + 11)}$$

$$p_E = \frac{\left(92\eta^2 + 510\eta + 139\right) t}{18(46\eta + 11)}$$

$$p_F = \frac{\left(128\eta^2 + 474\eta + 139\right) t}{18(46\eta + 11)}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(44\eta^2 + 171\eta + 70\right) t}{9(35\eta + 22)}$$

$$p_B = \frac{\left(19\eta^2 + 135\eta + 74\right) t}{9(35\eta + 22)}$$

$$p_C = \frac{4\left(4\eta^2 + 33\eta + 20\right) t}{9(35\eta + 22)}$$

$$p_D = \frac{\left(26\eta^2 + 153\eta + 106\right) t}{9(35\eta + 22)}$$

$$p_E = \frac{\left(88\eta^2 + 375\eta + 278\right) t}{18(35\eta + 22)}$$

$$p_F = \frac{\left(88\eta^2 + 411\eta + 242\right) t}{18(35\eta + 22)}.$$

These prices are consistent with $p_A < p_F$ and $p_E < p_F$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(46\eta^2 + 204\eta + 35\right)t}{9(46\eta + 11)}$$

$$p_B = \frac{\left(23\eta^2 + 168\eta + 37\right)t}{9(46\eta + 11)}$$

$$p_C = \frac{\left(23\eta^2 + 165\eta + 40\right)t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(46\eta^2 + 186\eta + 53\right)t}{9(46\eta + 11)}$$

$$p_E = \frac{\left(161\eta^2 + 441\eta + 139\right)t}{18(46\eta + 11)}$$

$$p_F = \frac{\left(161\eta^2 + 510\eta + 70\right)t}{18(46\eta + 11)}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(22\eta + 73)t}{171}$$

$$p_B = \frac{(10\eta + 66)t}{171}$$

$$p_C = \frac{5(9\eta + 67)t}{171}$$

$$p_D = \frac{4(16\eta + 79)t}{171}$$

$$p_E = \frac{(55\eta + 192)t}{342}$$

$$p_F = \frac{(78\eta + 169)t}{342}.$$

These prices are inconsistent with $p_E \geq p_F$ or $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (ii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_F > p_E > p_A > p_D > p_B > p_C.$$

Hence, the worst-off population member is the one who is just indifferent between E and F. This consumer's position is

$$x^*_{EF} = \frac{47\eta + 10}{420\eta + 264}.$$

Plugging this in, the person indifferent between E and F prefers to purchase the good if:

$$1 - p_E - x^*_{EF} t \geq 0$$

which is true if and only if:

$$\frac{-176\eta^2 t - 891\eta t + 1260\eta - 586t + 792}{1260\eta + 792} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if $-176\eta^2 + 369\eta + 206 \geq 0$, which is true for any $\eta \in [1, 2]$.

The equilibrium rents are

$$u^{\mathbf{BC},ADEF}(\eta) = \frac{\left(313\eta^4 + 4686\eta^3 + 20497\eta^2 + 20532\eta + 5956\right) Nt}{81(35\eta + 22)^2}$$

$$u^{BC,\mathbf{ADEF}}(\eta) = \frac{\left(11744\eta^4 + 103041\eta^3 + 278285\eta^2 + 246312\eta + 68900\right) Nt}{648(35\eta + 22)^2}.$$

E.1.4 $ABCD, EF$

There are four cases to consider:

(i) Suppose $p_A \leq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(29\eta^2 + 184\eta + 72\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_B = \frac{\left(193\eta^2 + 416\eta + 132\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_C = \frac{\left(241\eta^2 + 392\eta + 108\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_D = \frac{\left(101\eta^2 + 148\eta + 36\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_E = \frac{4\left(19\eta^2 + 32\eta + 6\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_F = \frac{2\left(29\eta^2 + 73\eta + 12\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}.$$

These prices are inconsistent with $p_A \leq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A \leq p_F$ and $p_E \leq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(16\eta^2 + 163\eta + 106\right) t}{3\left(80\eta^2 + 77\eta + 14\right)}$$

$$p_B = \frac{\left(148\eta^2 + 395\eta + 198\right) t}{6\left(80\eta^2 + 77\eta + 14\right)}$$

$$p_C = \frac{\left(184\eta^2 + 387\eta + 170\right) t}{6\left(80\eta^2 + 77\eta + 14\right)}$$

$$p_D = \frac{\left(70\eta^2 + 151\eta + 64\right) t}{3\left(80\eta^2 + 77\eta + 14\right)}$$

$$p_E = \frac{4\left(4\eta^2 + 35\eta + 18\right) t}{3\left(80\eta^2 + 77\eta + 14\right)}$$

$$p_F = \frac{2\left(16\eta^2 + 65\eta + 33\right) t}{3\left(80\eta^2 + 77\eta + 14\right)}.$$

These prices are inconsistent with $p_A \leq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E \leq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(58\eta^2 + 155\eta + 72\right)t}{3\left(40\eta^2 + 82\eta + 49\right)}$$

$$p_B = \frac{\left(152\eta^2 + 386\eta + 203\right)t}{6\left(40\eta^2 + 82\eta + 49\right)}$$

$$p_C = \frac{\left(148\eta^2 + 380\eta + 213\right)t}{6\left(40\eta^2 + 82\eta + 49\right)}$$

$$p_D = \frac{\left(52\eta^2 + 146\eta + 87\right)t}{3\left(40\eta^2 + 82\eta + 49\right)}$$

$$p_E = \frac{2\left(10\eta^2 + 61\eta + 43\right)t}{3\left(40\eta^2 + 82\eta + 49\right)}$$

$$p_F = \frac{4\left(10\eta^2 + 38\eta + 9\right)t}{3\left(40\eta^2 + 82\eta + 49\right)}.$$

These prices are consistent with $p_A \geq p_F$ and $p_E \leq p_F$ for any $\eta \in [1, 2]$, so this is a candidate for an equilibrium.

(iv) Suppose $p_A > p_F$ and $p_E > p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(11\eta + 74)t}{171}$$

$$p_B = \frac{(20\eta + 227)t}{342}$$

$$p_C = \frac{(18\eta + 229)t}{342}$$

$$p_D = \frac{(8\eta + 87)t}{171}$$

$$p_E = \frac{(14\eta + 62)t}{171}$$

$$p_F = \frac{(24\eta + 52)t}{171}.$$

These prices are inconsistent with $p_E > p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (iii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_B > p_C > p_A > p_D > p_F > p_E.$$

Hence, the worst-off population member is the one who is just indifferent between B and C. This consumer's position is

$$x^*_{BC} = x = \frac{36\eta^2 + 76\eta + 59}{480\eta^2 + 984\eta + 588}.$$

Plugging this in, the person indifferent between B and D prefers to purchase the good if:

$$1 - p_B - x^*_{BC} t \geq 0$$

which is true for any $\eta \in [1, 2]$ and $t \in (0, 1]$.

The equilibrium rents are

$$u^{\mathbf{ABCD},EF}(\eta) = \frac{2\left(300\eta^5 + 2580\eta^4 + 7681\eta^3 + 9017\eta^2 + 5015\eta + 1399\right) Nt}{9\left(40\eta^2 + 82\eta + 49\right)^2}$$

$$u^{ABCD,\mathbf{EF}}(\eta) = \frac{\left(13760\eta^4 + 72960\eta^3 + 136060\eta^2 + 103304\eta + 28057\right) Nt}{36\left(40\eta^2 + 82\eta + 49\right)^2}.$$

E.1.5 $AB, CDEF$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(29\eta^2 + 165\eta + 34\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_B = \frac{2\left(31\eta^2 + 69\eta + 14\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_C = \frac{\left(103\eta^2 + 150\eta + 32\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_D = \frac{\left(263\eta^2 + 390\eta + 88\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_E = \frac{\left(233\eta^2 + 408\eta + 100\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_F = \frac{\left(58\eta^2 + 177\eta + 50\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}.$$

These prices are consistent with $p_A \leq p_F$ and $p_E \geq p_F$ for any $\eta \in [1, 2]$, so this is a candidate for an equilibrium.

(ii) Suppose $p_A \leq p_F$ and $p_E \leq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{2\left(11\eta^2 + 69\eta + 34\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_B = \frac{\left(43\eta^2 + 129\eta + 56\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_C = \frac{(31\eta + 64)t}{9(11\eta + 8)}$$

$$p_D = \frac{\left(139\eta^2 + 426\eta + 176\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_E = \frac{\left(88\eta^2 + 453\eta + 200\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_F = \frac{(22\eta + 73)t}{9(11\eta + 8)}.$$

These prices are inconsistent with $p_E \leq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E \leq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{2\left(13\eta^2 + 84\eta + 17\right) t}{9(46\eta + 11)}$$

$$p_B = \frac{\left(20\eta^2 + 165\eta + 43\right) t}{9(46\eta + 11)}$$

$$p_C = \frac{\left(28\eta^2 + 186\eta + 71\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(92\eta^2 + 441\eta + 208\right)t}{18(46\eta + 11)}$$

$$p_E = \frac{\left(128\eta^2 + 372\eta + 241\right)t}{18(46\eta + 11)}$$

$$p_F = \frac{\left(64\eta^2 + 204\eta + 17\right)t}{9(46\eta + 11)}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A > p_F$ and $p_E > p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(11\eta + 65)t}{171}$$

$$p_B = \frac{(8\eta + 68)t}{171}$$

$$p_C = \frac{(10\eta + 85)t}{171}$$

$$p_D = \frac{(32\eta + 215)t}{342}$$

$$p_E = \frac{(44\eta + 203)t}{342}$$

$$p_F = \frac{(28\eta + 67)t}{171}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (i)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_D > p_E > p_C > p_F > p_B > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between D and

E. This consumer's position is

$$x_{DE}^{*} = x = \frac{36\eta^2 + 76\eta + 59}{480\eta^2 + 984\eta + 588}.$$

Plugging this in, the person indifferent between D and E prefers to purchase the good if:

$$1 - p_D - x_{DE}^{*} t \geq 0,$$

which is true if and only if:

$$\frac{\eta^2(1044 - 583t) + \eta(864 - 870t) - 200t + 144}{36\left(29\eta^2 + 24\eta + 4\right)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if $461\eta^2 - 6\eta - 56 \geq 0$, which is true for any $\eta \in [1, 2]$.

The equilibrium rents are

$$u^{\mathbf{AB},CDEF}(\eta) = \frac{\left(841\eta^5 + 14503\eta^4 + 44527\eta^3 + 34069\eta^2 + 9208\eta + 820\right) Nt}{162\left(29\eta^2 + 24\eta + 4\right)^2}$$

$$u^{AB,\mathbf{CDEF}}(\eta) = \frac{\left(6728\eta^5 + 71201\eta^4 + 146966\eta^3 + 100262\eta^2 + 26576\eta + 2408\right) Nt}{324\left(29\eta^2 + 24\eta + 4\right)^2}.$$

E.2 Local Transportation Costs

Without loss of generality, suppose the two factions start controlling A, B, C and D, E, F. To find the incremental returns, I first characterize equilibrium in thethe five scenarios: ABC, DEF, $ABCF, DE$, $BC, ADEF$, $AB, CDEF$, and $ABCD, EF$.

E.2.1 ABC, DEF

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 281\tau + 197\right) t}{18\left(2\tau^2 + 19\tau + 39\right)}$$

$$p_B = \frac{\left(106\tau^2 + 571\tau + 583\right) t}{36(\tau + 3)(2\tau + 13)}$$

$$p_C = \frac{\left(38\tau^2 + 233\tau + 269\right) t}{18(\tau+3)(2\tau+13)}$$

$$p_D = \frac{\left(34\tau^2 + 247\tau + 259\right) t}{18(\tau+3)(2\tau+13)}$$

$$p_E = \frac{(43\tau + 41)t}{36(\tau+3)}$$

$$p_F = \frac{\left(40\tau^2 + 259\tau + 241\right) t}{18(\tau+3)(2\tau+13)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_E > p_B > p_A > p_F > p_D > p_C$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from E and F and the citizen indifferent between buying from A and B.

The citizen indifferent between E and F is located at

$$x^*_{EF} = \frac{4\tau^3 + 36\tau^2 + 37\tau - 17}{24\tau^3 + 252\tau^2 + 696\tau + 468}.$$

We need the following:

$$1 - p_E - x^*_{EF} t \geq 0$$

which is true if

$$\frac{-98\tau^3 t + 72\tau^3 - 835\tau^2 t + 756\tau^2 - 1285\tau t + 2088\tau - 482t + 1404}{36(\tau+3)\left(2\tau^2 + 15\tau + 13\right)} \geq 0$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-26\tau^3 - 79\tau^2 + 803\tau + 922 \geq 0$, which is true for any $\tau \in [1, 2]$.

The citizen indifferent between A and B is located at

$$x^*_{AB} = \frac{47 - 2\tau}{48\tau + 312}.$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{-242\tau^2 t + 144\tau^2 - 1247\tau t + 1368\tau - 1211t + 2808}{72(\tau+3)(2\tau+13)}.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-98\tau^2 + 121\tau + 1597 \geq 0$, which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABC},DEF}(\tau) = \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2}$$

and

$$u^{ABC,\mathbf{DEF}}(\tau) = \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2}.$$

E.2.2 $ABCF, DE$

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 376\tau + 303\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_B = \frac{\left(212\tau^2 + 1351\tau + 1401\right) t}{36\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_C = \frac{19(2\tau+3)t}{9(4\tau+15)}$$

$$p_D = \frac{\left(68\tau^2 + 415\tau + 429\right) t}{18\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_E = \frac{\left(43\tau^2 + 239\tau + 174\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_F = \frac{(179\tau + 201)t}{36(4\tau+15)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_A > p_B > p_F > p_C > p_E > p_D$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from A and F and the citizen indifferent between buying from A and B.

The citizen indifferent between A and F is located at

$$x^*_{AF} = \frac{8\tau^3 + 47\tau^2 + 14\tau - 69}{48\tau^3 + 468\tau^2 + 1320\tau + 900}.$$

We need the following:

$$1 - p_A - x^*_{AF}t \geq 0$$

which is true if

$$\frac{-272\tau^3 t + 144\tau^3 - 1893\tau^2 t + 1404\tau^2 - 2758\tau t + 3960\tau - 1005t + 2700}{36(\tau+1)\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-128\tau^3 - 489\tau^2 + 1202\tau + 1695 \geq 0$, which is true for any $\tau \in [1, 2]$.

The citizen indifferent between A and B is located at

$$x^*_{AB} = \frac{-4\tau^2 + 19\tau + 213}{96\tau^2 + 840\tau + 1800}.$$

We need the following:

$$1 - p_A - x^*_{AB}t \geq 0$$

which is true if

$$\frac{-484\tau^2 t + 288\tau^2 - 3065\tau t + 2520\tau - 3063t + 5400}{72\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-196\tau^2 - 545\tau + 2337 \geq 0$, which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABCF},DE}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right)Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2}$$

and

$$u^{ABCF,\mathbf{DE}}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right)Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

E.2.3 $BC, ADEF$

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{(46\tau + 239)t}{99\tau + 414}$$

$$p_B = \frac{(85\tau + 371)t}{198\tau + 828}$$

$$p_C = \frac{(91\tau + 365)t}{198\tau + 828}$$

$$p_D = \frac{(64\tau + 221)t}{99\tau + 414}$$

$$p_E = \frac{(355\tau + 1127)t}{36(11\tau + 46)}$$

$$p_F = \frac{(605\tau + 2359)t}{72(11\tau + 46)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_E > p_F > p_D > p_A > p_C > p_B$. Hence, the worst-off consumer is the one indifferent between buying from E and F.

The consumer indifferent between E and F is located at

$$x^*_{EF} = \frac{44\tau^2 + 149\tau + 35}{264\tau^2 + 1368\tau + 1104}.$$

We need the following:

$$1 - p_E - x^*_{EF}t \geq 0$$

which is true if

$$\frac{\tau^2(792 - 842t) - 9\tau(379t - 456) - 2359t + 3312}{72(\tau + 1)(11\tau + 46)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, this condition holds if and only if $-50\tau^2 + 693\tau + 953 \geq 0$, which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$u^{BC,\mathbf{ADEF}}(\tau) = \frac{19\left(409\tau^2 + 3406\tau + 7129\right)Nt}{324(11\tau+46)^2}$$

and

$$u^{\mathbf{BC},ADEF}(\tau) = \frac{\left(109193\tau^3 + 995320\tau^2 + 2701885\tau + 1859858\right)Nt}{2592(\tau+1)(11\tau+46)^2}.$$

E.2.4 $ABCD, EF$

Assuming that demand is given by Equations 4 and 8, ta ing first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(87\tau^2 + 293\tau + 190\right)t}{9\left(4\tau^2 + 41\tau + 69\right)}$$

$$p_B = \frac{\left(78\tau^2 + 341\tau + 322\right)t}{9\left(4\tau^2 + 41\tau + 69\right)}$$

$$p_C = \frac{\left(126\tau^2 + 655\tau + 701\right)t}{18\left(4\tau^2 + 41\tau + 69\right)}$$

$$p_D = \frac{\left(84\tau^2 + 505\tau + 551\right)t}{18\left(4\tau^2 + 41\tau + 69\right)}$$

$$p_E = \frac{2\left(15\tau^2 + 116\tau + 97\right)t}{9\left(4\tau^2 + 41\tau + 69\right)}$$

$$p_F = \frac{\left(159\tau^2 + 880\tau + 785\right)t}{36\left(4\tau^2 + 41\tau + 69\right)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_B > p_C > p_A > p_D > p_F > p_E$. Hence, the worst-off consumer is either the one indifferent between buying from B and C or the one indifferent between A and F.

The consumer indifferent between B and C is located at

$$x^*_{BC} = \frac{-3\tau^2 + 16\tau + 44}{24\tau^2 + 246\tau + 414}.$$

We need the following:

$$1 - p_B - x^*_{BC}t \geq 0$$

which is true if
$$\frac{\left(147\tau^2+730\tau+776\right)t}{18\left(4\tau^2+41\tau+69\right)}\geq 0,$$
which clearly holds.

The consumer indifferent between A and F is located at
$$x^*_{AF}=\frac{24\tau^3+57\tau^2+122\tau+25}{144\tau^3+1620\tau^2+3960\tau+2484}.$$
We need the following:
$$1-p_A-x^*_{AF}t\geq 0$$
which is true if
$$\frac{\left(372\tau^3+1577\tau^2+2054\tau+785\right)t}{36(\tau+1)\left(4\tau^2+41\tau+69\right)}\geq 0,$$
which clearly holds.

The rents at these equilibrium prices are:
$$u^{ABCD,\mathbf{EF}}(\tau)=\frac{\left(3600\tau^5+72681\tau^4+455584\tau^3+1135438\tau^2+1200976\tau+458697\right)Nt}{648(\tau+1)\left(4\tau^2+41\tau+69\right)^2}$$
and
$$u^{\mathbf{ABCD},EF}(\tau)=\frac{\left(5022\tau^5+83100\tau^4+441587\tau^3+990053\tau^2+969443\tau+343923\right)Nt}{324(\tau+1)\left(4\tau^2+41\tau+69\right)^2}.$$

E.2.5 $AB, CDEF$

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:
$$p_A=\frac{\left(43\tau^2+239\tau+174\right)t}{9\left(4\tau^2+35\tau+75\right)}$$
$$p_B=\frac{\left(68\tau^2+415\tau+429\right)t}{18\left(4\tau^2+35\tau+75\right)}$$
$$p_C=\frac{19(2\tau+3)t}{9(4\tau+15)}$$
$$p_D=\frac{\left(212\tau^2+1351\tau+1401\right)t}{36\left(4\tau^2+35\tau+75\right)}$$

$$p_E = \frac{\left(62\tau^2 + 376\tau + 303\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_F = \frac{(179\tau + 201)t}{36(4\tau + 15)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_E > p_D > p_F > p_C > p_A > p_B$. Hence, the worst-off consumer is either the one indifferent between buying from E and D or the one indifferent between E and F.

The consumer indifferent between D and E is located at

$$x^*_{DE} = \frac{20\tau^2 + 121\tau + 87}{96\tau^2 + 840\tau + 1800}.$$

We need the following:

$$1 - p_D - x^*_{DE} t \geq 0$$

which is true if

$$\frac{\tau^2(288 - 484t) - 5\tau(613t - 504) - 3063t + 5400}{72\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check at $t = 1$ which clearly holds. At $t = 1$, this condition holds if and only if $-196\tau^2 - 545\tau + 2337 \geq 0$, which is true for any $\tau \in [1, 2]$.

The consumer indifferent between E and F is located at

$$x^*_{EF} = \frac{8\tau^3 + 47\tau^2 + 14\tau - 69}{48\tau^3 + 468\tau^2 + 1320\tau + 900}.$$

We need the following:

$$1 - p_E - x^*_{EF} t \geq 0$$

which is true if

$$\frac{\left(8\tau^3 + 47\tau^2 + 14\tau - 69\right) t}{12\left(4\tau^3 + 39\tau^2 + 110\tau + 75\right)} \geq 0,$$

which clearly holds for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$u^{AB,\mathbf{CDEF}}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right) Nt}{1296(\tau + 1)(\tau + 5)(4\tau + 15)^2}$$

and

$$u^{\mathbf{AB},CDEF}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right) Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

F Conflict Outcomes for Local Comparative Statics with Distant Territories

In the appendix, I characterize conflict outcome for when there are local shocks and distant territories C or D are vulnerable.

F.1 Market Size Shock at F, territory D vulnerable

Using the equilibrium rents calculated in Appendices E.1.1 and E.1.4, the incremental returns are:

$$\mathrm{IR}^{\text{pop}}_{\text{att},D}(\eta) = \frac{\left(-819200\eta^9+9585920\eta^8+81602432\eta^7+213151952\eta^6+247864928\eta^5+128843336\eta^4+14217824\eta^3-11967955\eta^2-4383712\eta-435350\right)Nt}{2592(8\eta^2+6\eta+1)^2(40\eta^2+82\eta+49)^2}$$

and

$$\mathrm{IR}^{\text{pop}}_{\text{def},D}(\eta) = \frac{\left(-2252800\eta^9-15361280\eta^8-34298240\eta^7+15677488\eta^6+144528352\eta^5+204315544\eta^4+136938448\eta^3+48046267\eta^2+8425048\eta+581498\right)Nt}{2592(8\eta^2+6\eta+1)^2(40\eta^2+82\eta+49)^2}.$$

Comparing, it is straightforward that the attacker's incremental return is higher. The conflict outcomes now follow from the analysis in the paper.

F.2 Market Size Shock at F, territory C vulnerable

Using the equilibrium rents calculated in Appendices E.1.1 and E.1.5, the incremental returns are:

$$\mathrm{IR}^{\text{pop}}_{\text{att},C}(\eta) = \frac{\left(3740564\eta^8+14688412\eta^7+23099109\eta^6+19710000\eta^5+10132194\eta^4+3230232\eta^3+624856\eta^2+67136\eta+3072\right)Nt}{1296(8\eta^2+6\eta+1)^2(29\eta^2+24\eta+4)^2}$$

and

$$\mathrm{IR}^{\text{pop}}_{\text{def},C}(\eta) = \frac{\left(2501164\eta^8+9618548\eta^7+16444995\eta^6+15483888\eta^5+8645802\eta^4+2932680\eta^3+593048\eta^2+65728\eta+3072\right)Nt}{1296(8\eta^2+6\eta+1)^2(29\eta^2+24\eta+4)^2}.$$

Comparing, it is straightforward that the attacker's incremental return is higher. The conflict outcomes now follow from the analysis in the paper.

F.3 Transportation Cost Shock at F, territory D vulnerable

Using the equilibrium rents calculated in Appendices E.2.1 and E.2.4, the incremental returns are:

$$\mathrm{IR}_{\mathrm{att},D}^{\mathrm{trans}}(\tau) = \frac{\left(24352\tau^7+694400\tau^6+7967340\tau^5+48379184\tau^4+172152361\tau^3+365544321\tau^2+418476195\tau+191491047\right)Nt}{1296(\tau+3)(2\tau+13)^2(4\tau^2+41\tau+69)^2}$$

and

$$\mathrm{IR}_{\mathrm{def},D}^{\mathrm{trans}}(\tau) = \frac{\left(21568\tau^7+724112\tau^6+10120788\tau^5+73390052\tau^4+287113309\tau^3+589602363\tau^2+598671711\tau+241601697\right)Nt}{2592(\tau+3)(2\tau+13)^2(4\tau^2+41\tau+69)^2}.$$

Comparing, it is straightforward that the defender's incremental return is higher. The conflict outcomes now follow from the analysis in the paper.

F.4 Transportation Cost Shock at F, territory C vulnerable

Using the equilibrium rents calculated in Appendices E.2.1 and E.2.5, the incremental returns are:

$$\mathrm{IR}_{\mathrm{att},C}^{\mathrm{trans}}(\tau) = \frac{\left(32832\tau^6+828464\tau^5+8394468\tau^4+43071096\tau^3+116188669\tau^2+154128630\tau+78932241\right)Nt}{2592(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}$$

and

$$\mathrm{IR}_{\mathrm{def},C}^{\mathrm{trans}}(\tau) = \frac{\left(16416\tau^6+369056\tau^5+3340356\tau^4+15762684\tau^3+41390347\tau^2+57437184\tau+31787757\right)Nt}{1296(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}.$$

Comparing, it is straightforward that the attacker's incremental return is higher. The conflict outcomes now follow from the analysis in the paper.