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Abstract

We examine how variation in motivations affect the resilience of movements to repression. Groups whose members are materially rather than psychologically motivated are less affected by both targeted and indiscriminate repression, but are also less able to turn early failures into future successes. A key distinction we draw is that material rewards are rival while psychological rewards are non-rival. Rivalry of material rewards introduces congestion externalities to the coordination problem of collective action: repression that decreases the likelihood of success also decreases participation, so that each participant's share of the potential rewards is larger, mitigating the impact of the repression. A government attempting to control a movement should focus on raising participation costs when motivations are psychological and on destroying rewards when motivations are material.

Keywords: Regime Change, Rebellion, Repression, Material Incentives, Psychological Incentives

At least since Gurr (1970)'s classic book, political scientists have been interested in the motivations that drive people to join social movements, protests, and rebellions. There is considerable heterogeneity in such motivations. In some movements, most members are motivated primarily by material considerations. In other movements, the primary motivation is ideological or psychological. For example, Weinstein (2007) and Humphreys and Weinstein (2008) find that rebel fighters in Sierra Leone were motivated by opportunities for looting, drug sales, and other material gains. In contrast, Wood (2003) finds that rebels in El Salvador were motivated by psychological rewards, ranging from vengeance to the opportunity to be "part of the making of history" (18–19). (See Blattman and Miguel (2010, 32–35) for a review of the literature.)

The question of motivations for collective action is foundational for a behavioral understanding of mobilization and the internal dynamics of groups (Ellis 1999; Weinstein 2007; Pearlman 2013). But motivations also matter for the strategic interaction between movements and governments. This is especially true with regard to questions surrounding government repression.

We examine how variation in motivations affect the resilience of movements to repression. Comparing groups whose members are psychologically versus materially motivated, we ask three key questions. First, against which type of group is targeted repression that increases the costs of participation more effective? Second, against which type of group is indiscriminate repression, directed against all members of society associated with the movement (whether mobilized or not), more effective? Third, which type of group is more likely to succeed following an early defeat by repressive forces, if that early defeat leaves behind a residual core of highly committed members?

The recent literature on the interaction of repression and motivations tends to focus on settings where repression, rather than effectively reducing mobilization, instead causes backlash. Backlash is, of course, an important theme quite generally in the repression literature (Francisco 1995; Wood 2003; Kalyvas 2006; Bueno de Mesquita and Dickson 2007; Martin 2007; Davenport 2007; Earl 2011). Many historical cases, dating all the way back to ancient times, exhibit backlash—consider, for instance the Maccabean Revolt in response to Seleucid repression in the 2nd century BCE (Bickerman 1962; Stern 1968), or the Second Fitna rebellions following the killing of Hussayn (the second Shia Imam) under the Umayyad Caliphate in the 7th century CE (Dakake 2007). And the empirical literature documents instances of backlash in contemporary conflicts from Northern Ireland (LaFree, Dugan, and Korte 2009) to Israel/Palestine (Dugan and Chenoweth 2012; Benmelech, Berrebi, and Klor 2015) to Iran (Rasler 1996) to Vietnam

(Kocher, Pepinsky, and Kalyvas 2011; Dell and Querubin 2018) to Turkey (Aytaç, Schiumerini, and Stokes 2018).

Explanations for backlash often hinge on a theory of motivations. For instance, Opp and Ruehl (1990) and Rasler (1996) emphasize the ways in which repression can create new motivations for collective action through disillusionment with the status quo political order. (See Balcells (2012) and Zhukov and Talibova (2018) for related evidence.) Aytaç, Schiumerini, and Stokes (2018) focus on a psychological mechanism that operates through moral and emotional outrage. Siqueira and Sandler (2007) and Dugan and Chenoweth (2012) highlight how repression may radicalize those opposed to the regime, especially when repressive activities come at the expense of other policies (e.g., public good provision) that tend to moderate public opinion.

But repression does not always cause backlash. There is substantial evidence that repression works to reduce armed mobilization in many settings (Tilly 1978; Kalyvas 2006; Davenport 2007; Downes 2008; Earl 2011; Tarrow 2011). Indeed, there is such evidence even for the use of indiscriminate repression in conflicts including Guatemala (Stoll 1993), Iraq (Condra and Shapiro 2012), Chechnya (Lyall 2009), and Ukraine (Rozenas and Zhukov 2019).

Despite this empirical evidence, significantly less theoretical attention has been paid to the interaction of various forms of repression and motivations in settings where repression is in fact expected to work. One partial exception is Toft and Zhukov's (2015) important study showing that counterinsurgency differentially affects Islamist and nationalist groups in the North Caucasus. However, despite studying two different types of groups, Toft and Zhukov do not focus on differences in the underlying motivations of group members. This is because, they argue, there is considerable scholarly disagreement about extent to which the Islamist groups they study are spiritually versus materially motivated. As such, their theoretical account of variation in the efficacy of repression focuses instead on the fact that Islamist organizations are less structurally dependent on support from a surrounding population than are nationalist organizations. (Limodio (2019) studies how increased access to funds differentially affects Sunni and non-Sunni terror organizations in Pakistan.)

Our focus is precisely on providing a theoretical account of how different types of motivations affect the efficacy of various forms of repression in a setting where repression is expected to work. We study two types of motivation: material and psychological. In our conceptualization, material and psychological motivations share an important feature and also differ in an important respect. The feature they share in common is that both are contingent on the success of the movement. The point of divergence is that material rewards are rival goods, while psychological rewards are non-rival.

We show that movements whose members are materially rather than psychologically motivated are less affected by repression, whether targeted or indiscriminate. However, movements whose members are psychologically rather than materially motivated are better able to turn early failures into future successes. These results are driven by the fact that material rewards are success-contingent and rivalrous, while psychological rewards are success-contingent but non-rivalrous. When rewards are material, as the movement becomes larger, success is more likely, but the rewards to each individual conditional on success are smaller. When rewards are psychological, as the movement becomes larger, success is more likely and rewards don't change. As such, the setting with psychological rewards features only strategic complements. But the setting with material rewards also has a force for strategic substitutes. Given the centrality of the conceptual distinction we make between material and psychological motivations, it is important to discuss the arguments for its validity.

The Nature of Material and Psychological Motivations As indicated above, we treat both psychological and material rewards as contingent on the success of the movement. But we treat material rewards as rival goods and psychological rewards as non-rival. Why these similarities and distinctions?

Our focus on success-contingent motivation is consistent with Rasler's (1996) "value expectancy" model, and her empirical evidence supports the associated implications (148). As Rasler (1996) argues: "Value-expectancy models assert that people will rebel if they become convinced that dissent will achieve the collective good (Klandermans 1984; Muller and Opp 1986; Finkel, Muller, and Opp 1989). If the value of the collective good (e.g., overthrow of the Shah's government) is combined with a high expectation of success, people are likely to participate in mass actions" (134).

Material benefits take various forms, including direct payments, protection, opportunities for looting, and promises of future economic spoils. Some of these, such as looting while fighting, may be enjoyed by participants even during the course of a failing campaign. But all are larger in a successful movement and many, such as rents associated with taking over the economy, are only available following success. The economic spoils available at the end of a successful rebellion are finite. They must be divided among the participants in the victorious movement. As such, it is natural to think that they are rival—the larger the movement, the less each individual participant expects to receive. This assumption is consistent with a large literature arguing that there is often conflict among rebels when the rebellion is materially motivated. (See Fjelde and Nilsson (2012) for a discussion.)

The nature of psychological benefits has been the subject of long debate. Early work emphasized purely expressive motives (Davies 1962; Geschwender 1967; Gurr 1970). But later studies showed that even psychologically motivated individuals account for the likelihood of success and the costs of participation when deciding whether to mobilize (Tilly 1978, 2008; McAdam 1999; Tarrow 2011).[1] In particular, movements with no prospect of success are unlikely to be sustainable because the costs of participation exceed the psychological benefits. Later studies confirmed this insight and developed a success-contingent conception of psychological and ideological rewards. In particular, Wood's (2003) notion of pleasure-in-agency captures psychological rewards associated with participating in a successful movement. Wood defines pleasure-in-agency as, "the positive effect associated with self-determination, autonomy, self-esteem, efficacy, and pride that come from the successful assertion of intention" (235). Based on extensive fieldwork and building on the historical and sociological literature, Wood found that agents motivated by psychological rewards not only account for the likelihood of success, but also act strategically: pleasure-in-agency is, "a frequency-based motivation: it depends on the likelihood of success, which in turn increases with the number participating (Schelling 1978; Hardin 1982)" (235–6). Such findings suggests that whether psychological rewards derive from ideology, the satisfaction of "being part of the making of history," justice, honor, or vengeance, the net benefit is positive only if the movement succeeds (Petersen 2001; Wood 2003; Morris and Shadmehr 2017; Pearlman 2018; Aytaç and Stokes 2019). By contrast with the material setting, the satisfaction from implementing an ideological vision, achieving justice, or being part of history is not diminished for being shared. As such, it is natural to think of psychological rewards as non-rival—as more people join the movement, the likelihood of success increases, with no associated diminution in individual rewards conditional on success.

The Theoretical Framework Capturing these ideas—especially the rivalrous nature of material rewards—requires an analysis with multiple people considering whether or not to mobilize.

[1] As Washington wrote to the Continental Congress in the 1770s, "The honor of making a brave defense does not seem to be a sufficient stimulus, when the success is very doubtful and the falling into the Enemy's hands probable" (Middlekauff 2005, 342).

This, in turn, gives rise to coordination considerations—whether one individual wishes to participate depends on her beliefs about how many others will participate. The dual presence of coordination concerns and congestion externalities (due to rivalrous material rewards) significantly complicates the strategic environment, precluding the application of standard models. Almost all models of protest and revolution feature pure coordination considerations, and no congestion externalities.[2] The complexity arises because there is a force for strategic complementarity (when more people mobilize, the chances of success are higher) and a force for strategic substitutes (when more people mobilize, the rewards of victory are smaller). Consequently, for example, as a citizen becomes more optimistic about the likelihood of regime change, her incentives to participate may, paradoxically, fall. The analysis of these competing forces and their interactions with repression requires a formal model that incorporates both forces in a tractable manner. Our goal is to provide such a model and analysis.

Our model consists of a continuum of agents who simultaneously decide whether to participate in a costly rebellion. The rebellion succeeds if the measure of rebels exceeds a threshold that captures the regime's ability to withstand popular uprising. This captures the idea that, even a state prepared to engage in considerable repression, will face real pressure if that repression fails to curtail popular unrest. As Erich Mielke, the head of East German Stasi, told Erich Honecker, the president of East Germany, in 1989: "we can't beat up hundreds of thousands of people" (Przeworski 1991, 64). We will sometimes refer to this threshold as *regime strength*, although it in face captures only one aspect of regime strength. For instance, we will treat the regime's exercise of strength through repression separately.

In the setting with psychological rewards, if the rebellion succeeds, each participant receives a given reward regardless of the number of participants. In contrast, in the setting with material rewards, if the rebellion succeeds, a given reward is divided equally among the participants. We normalize the total size of available rewards, so that if all agents participate and the rebellion succeeds, each agent's payoff is the same in both settings. The regime's strength is uncertain and agents receive noisy private signals about it. Upon receiving their private information, players simultaneously decide whether to rebel, the success or failure of the movement is determined, and the payoffs are received.

In the complete information benchmark in which regime strength is known, psychological

[2]An exception is Shadmehr (2019b), which studies the interactions between political stability and the economy. We make use of the results in that paper for our technical characterization of equilibrium.

and material reward settings have similar properties. Both feature multiple equilibria, one in which no one rebels and one in which everyone rebels. These equilibria are insensitive to variation in the environment, such as increased repression (Proposition 1).

The assumption of complete information is, of course, substantively unrealistic. In the actual settings the model is intended to represent, citizens face genuine uncertainty about the strength of the regime and, thus, the likelihood of success. As such, we introduce such uncertainty, giving players private beliefs about regime strength, which induces strategic uncertainty about the level of mobilization. The introduction of such strategic uncertainty selects an equilibrium and shows how equilibrium behavior is differentially responsive to various forms of repression depending on the motivations (Proposition 2).

Increases in targeted repression—modeled as an increase in the cost of mobilization—reduce direct incentives to rebel in both settings, but the strategic effect differs across them (Proposition 3). Realizing that higher mobilization costs mean that others are less likely to rebel, an individual's incentives to mobilize fall even further in response to repression in the setting with psychological rewards. But this strategic effect is weaker in the material rewards setting and may even counteract the direct effect because a smaller rebellion size implies higher rewards for participants if the rebellion does succeed. Consequently, targeted repression is less effective against movements whose members are materially motivated than against movements whose members are psychologically motivated.

Increasing indiscriminate repression—modeled as a reduction in the size of the population of potential recruits—is also more effective against psychologically motivated groups (Proposition 4). In both settings, a reduced pool of potential recruits has a direct effect of reducing expected mobilization. And, again, in the psychological motivations setting, the strategic effect pushes in the same direction—reducing incentives to mobilize—while in the material rewards setting there is a strategic effect that pushes in the opposite direction since the reduction in expected mobilization increases an individual's reward conditional on success. Hence, indiscriminate repression is also expected to be less effective against movements whose members are materially motivated than against movements whose members are psychologically motivated.

The above results speak to interaction between repression and motivations in a static setting. But rebellion and repression often unfold over time. A movement that is successfully repressed at first may resurface again when another opportunity arises (McAdam, Tarrow, Tilly 2001; Tarrow 2011). And, indeed, early repressive successes often generate a core of committed members motivated by a sense of injustice, vengeance, or camaraderie. For instance, Lawrence (2017) documents how many of the first movers in Morocco's February 20th movement were members of families that had been repressed in previous instances of anti-regime collective action. Similar patterns have been observed in a variety of places, including El Salvador (Wood 2003), Iran (Rasler 1996; Shadmehr 2017), Hong Kong (Bursztyn et al. 2019), and Syria (Pearlman 2020).

The presence of a committed core increases the likelihood of success in both material and psychological rewards settings because the subsequent rebellion is sure to have the active support of the committed group. However, this effect is weaker with material rewards because uncommitted citizens recognize that material rewards have a tighter upper bound because the size of this subsequent rebellion is at least as large as the size of the committed core with whom the rewards will be shared. Consequently, conditional on suffering a repressive success that also creates a committed core, rebel movements whose participants are psychologically motivated are more likely to succeed than are rebel movements whose participates are materially motivated (Proposition 5).

1 Model and Analysis

There is a continuum of citizens of size $a > 0$, indexed by $i \in [0, a]$. Citizens simultaneously decide whether to join a rebel movement. The rebellion succeeds if and only if the size of the rebel movement, m, exceeds the state of the world, θ, which captures the strength of the status quo regime.

The payoff of a citizen who does not rebel is normalized to 0. If a citizen rebels, he pays a cost of $c \in (0, 1)$. If the rebellion succeeds, a citizen who participated in the rebellion receives a payoff u^j, $j = p, s$, where u^p is the reward in the setting with *psychological* rewards, and u^m is the reward in the setting with *material* rewards. Psychological rewards are normalized to 1, and material rewards are normalized to $\frac{a}{m}$, so that if the rebellion succeeds, the total available rewards in both settings is a. Figure 1 represents the payoffs.

The state of world is uncertain, and citizens share a common prior that θ is distributed on $\mathbb{R}$ according to an improper uniform distribution. Each citizen i receives a noisy private signals $x_i = \theta + \sigma\epsilon_i$, where θ and ϵ_is are distribute independently, with $\epsilon_i \sim F$ and the corresponding pdf f. We assume f is log-concave with full support on $\mathbb{R}$.

	outcome	
	$m > \theta$	$m \leq \theta$
rebel	$1 - c$	$-c$
not rebel	0	0

psychological

	outcome	
	$m > \theta$	$m \leq \theta$
rebel	$\frac{a}{m} - c$	$-c$
not rebel	0	0

material

Figure 1: **Psychological versus Material Rewards.** The size of the population is a, the size of rebel movement is $m \leq a$, the cost of participation is c, and the regime's strength is θ. The left panel captures movements with psychological rewards: net rewards from participation do not depend on how many participate. The right panel captures movements with material rewards: net rewards from participation fall with more participation because participants must share the spoils.

1.1 Complete Information Benchmark

We begin with the complete information benchmark in which the regime's strength θ is known. (All proofs are in the appendix.)

Proposition 1 *The setting with psychological rewards and the setting with material rewards both have the same pure strategy equilibria:*

- *If $\theta \geq a$, there is a unique equilibrium in which no one rebels, the regime survives, and each citizen receives* 0.
- *If $\theta < 0$, there is a unique equilibrium in which everyone rebels, the regime collapses, and everyone receives* 1.
- *In between, both equilibria co-exist.*

Proposition 1 implies that the settings with psychological and material rewards generate the same outcomes. However, this complete information setting is misleading. Of course, it abstracts from information frictions that exist in the real world. Moreover, it has two problematic properties: (1) there are multiple equilibria, which makes empirical prediction difficult; and (2) equilibrium outcomes are insensitive to parameters of the model like the costs of rebellion. The introduction of incomplete information addresses both issues.

1.2 Equilibrium

We now analyze our main, incomplete information model. The left panel in Figure 1 is the quintessential regime change game (Morris and Shin 1998, 2003; Angeletos et al. 2007).[3] In it, equilibrium is characterized by two thresholds (x^p, θ^p), so that citizens with signal $x_i < x^p$ rebel, and the regime collapses if and only if $\theta < \theta^p$. These threshold are determined by the indifference (optimality) and belief consistency conditions:

$$a \Pr(x_i < x^p | \theta^p) = \theta^p \text{ (belief consistency)} \quad \text{and} \quad \Pr(\theta < \theta^p | x_i = x^p) = c \text{ (indifference)}. \tag{1}$$

Because each citizen rebels whenever her signal of the regime's strength is below a threshold, for any given regime strength θ, the aggregate size of the rebellion is $a \Pr(x_i < x^p|\theta)$. Naturally, the size of the rebellion is decreasing in the regime's strength, implying that the regime collapses below a threshold of regime strength and survives above it. Thus, that critical threshold (which we call θ^p) is exactly the size of the rebellion at that critical threshold.

How do we find this critical rebellion size? Because a citizen rebels whenever her belief about the likelihood of success is larger than the cost of rebelling, to find the size of the rebellion at the critical threshold we need to know the distribution of these beliefs at that critical threshold. As Shadmehr (2019a) discusses in detail, when there is no prior knowledge about θ,[4] the distribution of these beliefs about the likelihood of success at the critical threshold is uniformly distributed on $[0, 1]$ among citizens. Thus, the size of the rebellion is the population size a times probability that a random citizen's belief is above the rebellion cost: $a(1 - c)$. That is,

$$\theta^p = a(1 - c). \tag{2}$$

The nature of strategic interactions is a pure coordination problem. The game is a standard global game of regime change, where the actions of citizens are always strategic complements: when one citizen believes that others are more likely to rebel, her incentives to join the rebellion increase because the rewards remain the same, but the likelihood of success increases.

In contrast, the game in right panel of Figure 1 is not a pure coordination game. In this game, when a citizen believes that others are more likely to rebel, her incentives to protest

[3] Papers featuring variations of this game include Boix and Svolik (2013), Edmond (2013), Casper and Tyson (2014), Loeper et al. (2014), Chen et al. (2016), Rundlett and Svolik (2016), Tyson and Smith (2018), and Shadmehr (2019b).

[4] For example, when citizens share a prior that θ is distributed accordingly to an improper uniform distribution on $\mathbb{R}$, or when the noise in private signals is vanishingly small.

may fall because, although success is more likely, the limited rewards from that success will be shared among a larger group, so that each participant will expect to receive less reward conditional on success. That is, the game does not feature global strategic complements due to congestion externalities. In particular, for a given level of regime strength, θ, the net payoff from revolting versus not revolting is:

$$\mathbf{1}_{\{\theta<m\}} \cdot \frac{a}{m} - c.$$

This net payoff is non-monotone in the size of the rebellion m. It jumps up from 0 to $\frac{a}{m} - c$ at $m = \theta$ (the threshold at which regime change succeeds), but then falls smoothly to $1 - c$ as more people join the movement. Therefore, the best response to a monotone strategy is not monotone in general, and monotone equilibria may not exist. The source of this complication, relative to the psychological rewards setting, is that the expected rewards do not boil down to the likelihood of success, because higher chances of success also imply a larger rebellion size, which in turn, implies a smaller reward for each participant. That is, when a citizen receives a lower signal, she updates that the regime is weaker and the size of the rebellion larger. This updating increases her assessment of the chances of success, but reduces her assessment of the reward conditional on that success (the citizen updates $\mathbf{1}_{\{\theta<m\}}$ upward, but $\frac{a}{m}$ downward). Despite this non-monotonicity, Proposition 2 shows that our assumptions are enough to deliver the existence and uniqueness of symmetric monotone equilibria.[5]

Proposition 2 *The setting with psychological rewards has a unique equilibrium in which the rebellion succeeds if and only if the strength of the regime is below a threshold $\theta^p = a(1-c)$. The setting with material rewards has a unique equilibrium in which the rebellion succeeds if and only if the strength of the regime is below a threshold $\theta^m = ae^{-c}$.*

Proposition 2 implies that $\theta^m > \theta^p$. Because total rewards in the material setting are divided among rebel participants, and some citizens always choose not to rebel in equilibrium due to information frictions, the equilibrium incentives are stronger in the material rewards setting ($1 < a/m$ for $m \in (0, a)$).

With these equilibrium characterizations in hand, we can turn to our main topic of interest—the differential efficacy of various forms of repression against materially versus psychologically motivated rebel groups. But, before doing so, it is worth commenting on some features of our model.

[5]See Morris and Shin (2003, 68-70) and Shadmehr (2019b) for technical discussions.

1.3 Comments on the Model

Several natural questions arise from our basic set-up. The first is what happens if people are motivated by some mix of psychological and material motivations. We analyze this question in Online Appendix A, showing that the results in the mixed case lie in between the results for the pure material and pure psychological cases we consider in the main text.

The second has to do with robustness to the assumption of an improper uniform prior. We make that assumption, which is standard in the global games literature, as a relatively simple way to introduce strategic uncertainty while allowing us to focus on the question of interest—the effects of different types of motivation. We are not focused, here, on the effects of information per se. That said, in Online Appendix B, we show that the results are robust to other informational assumptions. In particular, we show that the same results obtain for any smooth prior in the limit when the noise becomes vanishingly small. We then provide numerical examples for a standard normal prior for both the case of a uniform distribution of noise and a standard normal distribution of noise. Finally, we provide additional numerical examples for the effect of a public signal about the strength of the regime (θ) in both settings with psychological and material rewards.

The third has to do with whether our results are sensitive to the normalization that total material and psychological rewards are equal at full participation and that, therefore, individual psychological rewards are less than individual material rewards for less than full participation. To address this concern, in Online Appendix C we show that our results are robust to a variant of the model where individual material rewards are given by ka/m, for $k > 0$ and $c/k \in (0, 1)$. In particular, Propositions 4 hold for any such (c, k). Proposition 3 holds for any $k \geq 1$ and for $k < 1$ as long as c is sufficiently large. And Proposition 5 holds for any $k \geq 1$ and for $k < 1$ as long as a and c are sufficiently large.

Finally, it is worth commenting on a few other features of payoffs in our model.

We have assumed that rewards are contingent on participating. This is distinct from rewards that are gained by every citizen if the regime falls—in the language of Olson (1965) and Tullock (1971), our rewards are selective/private benefits. As we argued extensively in the Introduction, we think the idea of participation-contingent rewards are substantively appropriate in both our material and psychological rewards settings. But it is also worth noting that our results are robust to adding rewards that are not participation contingent. In particular, since our model has a continuum of individuals, each individual regards their personal contribution to the probability of success as negligible. This implies that any reward (or cost) that does not depend on whether a person participates cannot affect that person's participation decisions. Hence, introducing additional rewards that are not contingent on participation would not alter our results.

It is also worth noting that we do not directly include costs that a citizen might suffer should she fail to participate in a rebellion that ultimately succeeds. Such costs are, of course, quite substantively plausible. But, notice, success-contingent costs associated with not participating are mathematically equivalent (with opposite sign) to success-contingent benefits associated with participating. So our model captures the substantive effects of such costs, without adding an additional parameter to directly represent them.

2 Targeted Repression

We represent the idea of an increase in targeted repression with an increase in the cost of rebellion, c. This corresponds with the standard conception of state repression as any action by the state "which raises the contender's cost of collective action" (Tilly 1978, 100; see also Davenport 2000, 5–7). Of course, repression may raise both a citizen's direct costs of rebelling and a citizen's direct benefit from rebelling due to a sense of injustice or a desire for vengeance (Wood 2003; Davenport 2007; Earl 2011; Siegel 2011; Lawrence 2017; Pearlman 2018; Aytaç and Stokes 2019; Shadmehr and Boleslavsky 2019). In our model, the cost c is, in fact, the ratio of the costs and benefits of rebellion. Representing increased repression with an increase in c means that even though both the numerator and the denominator may rise, we assume the direct cost-benefit ratio is increasing. This is consistent with the standard view of higher repression as reducing political opportunities (Tilly 1978, 2006; McAdam 1999; Davenport 2007; Earl 2011; Tarrow 2011).

Proposition 2 immediately implies:

Proposition 3 *The equilibrium regime change threshold is less responsive to repression in the material rewards setting than in the psychological rewards setting:* $\frac{\partial \theta^p}{\partial c} < \frac{\partial \theta^m}{\partial c} < 0$.

Proposition 3 shows how the likelihood of success in differently motivated groups responds to variation in targeted repression. (Notice, this result could also be interpreted in terms of, say, the economic opportunity costs of mobilization.) The direct effect of higher rebellion costs is to reduce incentives to protest in both settings. But there is also a strategic effect: a citizen recognizes that higher costs of mobilization mean that others have less incentives to rebel, and adjusts her behavior accordingly. In the psychological rewards setting, this further reduces the incentives to rebel because the likelihood of success is lower. This strategic effect in weaker in the material rewards setting and may off-set parts of the direct effect (if actions are strategic substitutes at equilibrium). The reason is that even though the likelihood of success is lower, the size of the rebellion is also smaller, so that if the rebellion succeeds each participant receives a larger reward. Due to this strategic effect, the likelihood of success in the material rewards setting is less sensitive to increases in the direct costs of rebellion. Thus, targeted repression is less effective against groups whose members are materially motivated.

3 Indiscriminate Repression

We represent indiscriminate repression as a decrease in the supply of potential recruits. In this sense, we think of indiscriminate repression as corresponding to both government repression that kills potential rebel supporters as well as the use of concentration camps or forced population resettlement (Azam and Hoeffler 2002; Valentino, Huth, and Balch-Lindsay 2004). And, indeed, the use of forced population transfer to reduce the pool of potential supporters for rebellion has been a feature of conflicts ranging from the ancient world (e.g., the practice was used frequently by the Assyrian empire) to more recent conflicts in places such as Bengal, Darfur, Kosovo, and Syria.[6]

Suppose that a population of size $b \in (0, a)$ is removed from the original population of size $a > 0$.[7] Label the new equilibrium regime change thresholds θ_r^j, $j \in \{p, m\}$.

Proposition 4 *With the removal of a population of size $b \in (0, a)$, the equilibrium regime change thresholds are $\theta_r^p = (a-b)(1-c)$ and $\theta_r^m = (a-b)e^{-c\frac{a-b}{a}}$. Decreasing the pool of potential recruits always reduces the likelihood of regime change, but the percentage change is larger in the psychological than in the material rewards setting: $\frac{\theta_r^p - \theta^p}{\theta^p} < \frac{\theta_r^m - \theta^m}{\theta^m} < 0$.*

Diminishing the pool of potential recruits reduces the potential size of the rebellion. In the psychological rewards setting, the direct and strategic effects decrease incentives to rebel be-

[6]For a discussion of the ancient world see Oded (1979). For more contemporary conflicts, see, among many others, Daly (2007), Downes (2008), Staniland (2009), Balcells and Stanton (Forthcoming).

[7]In Proposition 3 we stated the result for a marginal change in c. Here we state the result for a discrete change in the size of the pool of potential recruits. A discrete change seems more natural for the application.

cause a smaller rebellion decreases the likelihood of success. However, in the material rewards setting, there are also congestion externalities that pull in the opposite direction. A smaller rebellion means that the rewards of success will be divided among a smaller group. Therefore, the effect of indiscriminate repression is smaller in the material rewards setting.

Increasing rebellion costs and decreasing the size of the recruitment pool work through similar channels. But there is a subtle difference. With targeted repression that increases mobilization costs, the congestion externalities associated with material rewards only appear in the strategic effect. But with indiscriminate repression that shrinks the recruitment pool, the congestion externalities appear in both the direct and strategic effects. Nonetheless, overall, the same basic logic that makes settings with material rewards less sensitive to targeted repression also makes them less sensitive to indiscriminate repression.

4 Repression and a Committed Core

Our analysis thus far has focused on the differential effect of repression on movements with material versus psychological motivations in a static setting. We showed that materially inspired movements are less affected by both targeted and indiscriminate repression. We now ask a more dynamic question: which type of movement is more resilient to repression in the long-run, given that even successful repression often leaves behind a committed core that will attempt to resurrect the movement in the future.

Many movements do not succeed or fail in a single episode. A movement that initially appears to have been defeated may resurface later when another political opportunity arises (McAdam, Tarrow, Tilly 2001; Tarrow 2011). Moreover, in such instances, the experience of earlier repression often creates a core of deeply committed participants. For instance, Rasler (1996) highlights how, in the short-run, government repression appears to have succeeded in putting down the protests that preceded the Iranian revolution, but that in the longer-run the movement inspired by these protests rose back up on the foundation built by the committed core. Wood (2003) and Lawrence (2017) emphasize how the desire for justice or vengeance can create such a committed core, focusing on the cases of El Salvador and Morocco, respectively. Bursztyn et al. (2019) and Pearlman (2020) highlight the ways in which social ties can contribute to the creation of a committed core, with a focus on the cases of Hong Kong and Syria, respectively. (See also Diani and McAdam (2003)).

To study how early failure that leaves in place a committed core affects the ultimate likelihood of success, we extend the model to two periods, and normalize the population size to 1. The stage game in the first period is identical to the previous setting. If the rebellion succeeds, the game ends. However, if the rebellion fails in the first period, in the second period citizens again play a similar regime change game. However, now there is a committed core: a fraction $1-a \in (0,1)$ of citizens who will surely participate in the second period rebellion. Thus, there are two differences between periods 1 and 2: in the second period, a fraction $1-a$ of citizens are committed to the rebellion and citizens have the additional, common knowledge that the regime survived the first period. To ease exposition, we focus on Normal distributions of noise so that $F = N(0,1)$.

In period 2, each citizen has three pieces of information: her signal from the first period, her signal from the second period, and the fact that the regime has survived. Because conditional expectations of normally distributed variables are linear, a citizen's private information in period 2 is effectively the average of her private signals in periods 1 and 2. Let x_2 be that average. We refer to this average signal as a citizen's private signal in period 2.

As before, we focus on symmetric monotone equilibria. In period 1, a citizen rebels if and only if her private signal is below a threshold x_1^*. In period 2, a fraction $1-a$ of citizens will be committed and rebel, and a citizen from the remaining group rebels if and only if her private signal x_2 is below a threshold x_2^*. As in the static setting, there is no equilibrium in which a citizen always revolts: $x_t^* < \infty$. If the regime survives the first period, this implies $\theta > 0$. Thus, there could be an equilibrium in which $x_2^* = -\infty$, and only the fraction $1-a$ of (committed) citizens rebel. We focus on the interesting case of finite cutoff equilibria, so that $x_2^* \in \mathbb{R}$.

Because a single citizen's action does not change the outcome (each individual is too small to make a non-negligible difference), in the first period the equilibrium behavior of citizens is the same as in the static game. Let θ_t^j, $j \in \{p, m\}$, $t = 1, 2$, be the period t equilibrium regime change threshold in the settings with psychological ($j = p$) and material ($j = m$) rewards, respectively. Let x_t^p and x_t^m be the corresponding equilibrium citizen cutoffs. From our earlier analysis, $\theta_1^p = 1 - c$ and $\theta_1^m = e^{-c}$. In the second period, in the setting with material rewards, any pair of cutoffs (θ_2^m, x_2^m) that satisfy the following belief consistency and indifference conditions constitute an equilibrium:

$$\theta_2^m = (1-a) + a\Pr(x_i < x_2^m | \theta_2^m) \tag{3}$$

$$c = E\left[\left.\frac{\mathbf{1}_{\{\theta<\theta_2^m\}}}{(1-a)+a\Pr(x_j < x_2^m)}\right| x_i = x_2^m, \theta > \theta_1^m\right] \tag{4}$$

In the second period, in the setting with psychological rewards, any (θ_2^p, x_2^p) that satisfy the following conditions constitute an equilibrium:

$$\theta_2^p = (1-a) + a\Pr(x_i < x_2^p | \theta_2^p) \tag{5}$$

$$c = \Pr(\theta < \theta_2^p | x_i = x_2^p, \theta > \theta_1^p) \tag{6}$$

These equilibrium conditions reflect the two differences between period 2 and period 1. The information content of the regime's survival is reflected in conditioning on $\theta > \theta_1^j$, $j \in \{p, m\}$, in the indifference conditions. The emergence of a committed rebel core is represented by the term $1 - a$ in the belief consistency conditions.

It is bad news for the rebels that the regime survived the first period, they've learned that $\Pr(\theta < \theta_1^j) = 0$, $j \in \{p, m\}$. This may prevent rebellion in period 2 altogether (finite-cutoff equilibria may not exist). But if citizens' private information is sufficiently precise, they effectively discard the relatively imprecise information that $\theta \geq \theta_1^j$, $j \in \{p, m\}$: compared to their precise private information, this public information receives little weight in their Bayesian updating. Cross-period informational linkages have been studied elsewhere in the literature (Angeletos et al. 2007). To focus on the new insight that the effect of a committed core depends on motivations, in our theoretical results, we abstract away from the informational linkage across periods—letting the noise in the second period's private signals become vanishingly small. We then provide numerical examples with information linkages (i.e., learning) between periods and discuss the effect of informational linkage on the dynamic.[8]

[8]More generally, the formal literature has focused on various aspects of the dynamics of protest, while abstracting from others. For example, some papers focus on different aspects of signaling and coordination (Lohmann 1994; Bueno de Mesquita 2010; Loeper et al. 2014; Shadmehr and Bernhardt 2019; Barbera and Jackson 2020; Chen and Suen 2020), while others focus on the interactions between repression and dissent (Shadmehr and Boleslavsky 2019; Gibilisco 2020).

Proposition 5 *Suppose the noise in the second period's private signals becomes vanishingly small, and we focus on the largest equilibrium. Conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting.*

The logic is as follows. A failure in the first attempt creates a committed core. This group of committed participants increases the likelihood of success in both settings, but the effect is weaker in the material rewards setting because, conditional on success, the group that will share the rewards is surely larger than the size of the committed members. Combining Propositions 2 and 5 implies that movements with psychological rewards are less likely to succeed in the first period, but conditional on a failure in the first period, they are more likely to succeed than movements with material rewards. This result resonates with the finding in Shadmehr and Bernhardt (2019) that it is more difficult for a movement to begin organically (without a revolutionary vanguard); but movements that begin organically are more likely to succeed.

In this analysis, because citizens have very precise private information in the second period, they do not need to rely on the informational content of the failure in the first period. Consequently, similar results hold if the regime's strength is independent across periods, an assumption that fits settings in which sufficient time has passed since the first revolt. But suppose there are genuine informational linkages across periods, can our findings continue to hold?

To see the forces at work once information linkages are reintroduced, recall that regime change is more likely in the material rewards setting in the first period ($\theta_1^m > \theta_1^p$). Therefore, upon observing that the regime has survived the rebellion, citizens in the material rewards settings infer that the regime is stronger than citizens in the psychological rewards settings do. That is, the direct informational effect of early failures makes citizens more pessimistic about the likelihood of success in the material rewards settings than in the psychological rewards setting, reinforcing the effect of committed core. Hence, there is reason for optimism that the overall finding that rebellions characterized by psychological motivations will be more resilient following early failures is robust. Below we show computational results consistent with that intuition.

Lemma 1 in the proof of Proposition 5 shows how equilibrium regime change thresholds can be calculated away from the limit of vanishingly small noise. Suppose $a = 0.8$ and $c = 0.2$, so that $\theta_1^p = 1 - c = 0.8$ and $\theta_1^m = e^{-c} \approx 0.82$. Moreover, suppose the noise is normally distributed with the standard deviation of the average signal being σ. When $\sigma = 0.01$, we have: $\theta_2^p \approx 0.84$ and $\theta_2^m \approx 0.85$. Thus, consistent with Proposition 5, $\theta_2^p - \theta_1^p \approx 0.04 > \theta_2^m - \theta_1^m \approx 0.03$—the presence of a committed core has a larger impact in the setting with psychological rewards. Conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting.

Now, suppose we increases the noise to $\sigma = 0.02$, further moving away from the limiting case of vanishingly small noise. We still have $\theta_1^p = 1 - c = 0.8$ and $\theta_1^m = e^{-c} \approx 0.82$. In second period of the psychological rewards setting, we have $\theta_2^p \approx 0.838 > \theta_1^p$ and the revolution might succeed in the second period after failing in the first period. By contrast, in the second period of the material rewards setting, the likelihood of success is 0. This case is a stark example of our finding that conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting.

Our findings also do not depend critically on the assumption of a uniform prior. To see this, consider an example with a normal prior about θ.[9] Suppose that $a = 0.6$, $c = 0.7$, $\theta \sim N(0, 2)$, and that noise is distributed iid across citizens and periods according to $N(0, 0.25)$. Now, $\theta_1^p \approx 0.305$ and $\theta_1^m \approx 0.505$. As in our previous example, there is still a finite threshold in the psychological motivations setting, $\theta_2^p \approx 0.511$, and a positive probability of the revolution succeeding in the second period. But in the material motivations case, the revolution will surely fail in the second period, again providing a stark illustration of our result. Thus, in all these examples, allowing for an information linkage across periods strengthens the result that conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting.

5 Conclusion

We explored how the efficacy of different forms of repression vary depending on the motivations for participation in collective action. While movements with material rewards are less affected by both targeted and indiscriminate repression, movements with psychological rewards are more resilient to repressive efforts that result in early failures but leave behind a committed core. A key ingredient of the underlying logic is the rivalry of material rewards versus the non-rivalry of psychological rewards—the other key ingredient is strategic uncertainty as

[9]Generally, the information that the regime has survived is like a public signal about the regime's strength. This public signal changes the common prior between periods. Given this new common prior, the analysis of the second period is closely related to that of the static game, but with a general prior. In Online Appendix B, we provide an analysis of the static game with a general prior.

Propositions 1 and 2 reveal. The effects of these motivations is complicated by the presence of coordination concerns among citizens. We show that, because material rewards must be shared upon success, repression that decreases participation (e.g., increased costs or a diminished pool of potential recruits) have less influence on movements with material motives: such changes decrease the likelihood of success, but by decreasing participation they also increase the rewards of success in movements with material motives. Conversely, changes that increase the number of participants (like the presence of a committed core) are less beneficial to movements with material motives: they increase the likelihood of success, but this effect is partly counteracted with material motives because higher participation means that the rewards of success will be shared among a larger group.

These insights have policy implications. Policy makers should be more concerned about the existence of a committed core or vanguard when confronting movements with psychological and ideological motives rather than material motives. And, for this reason, they should be more cautious about the long-run efficacy of early victories when facing a group whose members are psychologically or ideologically motivated, as they are more resilient to such early failures than are movements using primarily material motives. By contrast, policy makers should recognize that movements motivated by material reward are more resilient to both targeted and indiscriminate repression. Combining these observations suggests the optimal policy that aims to reduce the chances of success are qualitatively different across movements whose members are differentially motivated. When dealing with movements whose members are motivated by psychological or ideological rewards, the focus should be on the raising the costs of participation. In contrast, in dealing with movements whose members are materially motivated, it might be more effective to focus on reducing the material rewards that participants hope to gain should the movement succeed.

Our analysis also suggests several avenues for future research. First, we introduced incomplete information into our model as a form of equilibrium selection, which allowed us to focus on how the efficacy of different types of repression depends on motivations. But we largely abstracted away from the substantive effects of information itself. Future work might explore how different sources of uncertainty interact with motivations and repression. Second, our focus was on how mobilization decisions respond to different forms of repression. But we did not model endogenous choices of repressive strategy by the regime. Understanding how regimes choose repressive strategies is an important question. Our results certainly have implications for such choices—suggesting that one input to the government's decision involves the motivations of the group it faces. But integrating these ideas into the broader literature on repressive strategies, and thinking about dynamic feedback between repressive strategies and mobilization, awaits future research. Finally, our results suggest hypotheses—about different forms of repression having heterogeneous effects depending on motivations—which we hope will motivate future empirical work.

Appendix: Proofs

Proof of Proposition 1: When $\theta < a$, the regime collapses if almost all citizens rebel, so that $m = a$. Thus, when $\theta < a$, a citizen rebels if she believes that almost all others will rebel, because her payoff from rebelling will be $1 - c > 0$. When $\theta \geq 0$, the regime survives if almost no citizen rebels, so that $m = 0$. Thus, when $\theta \geq 0$, a citizen does not rebel if she believes that almost no other citizen will rebel, because her payoff from rebelling will be $-c < 0$. $\square$

Proof of Proposition 2: We look for symmetric monotone equilibria in which a citizen rebels if and only if her signal is below a finite threshold $x_i < x^m$. A monotone strategy implies that the regime collapses if and only if $\theta < \theta^m$, where

$$a \Pr(x_i < x^m|\theta^m) = \theta^m \quad \text{(belief consistency)}. \tag{7}$$

A citizen with signal x_i rebels if and only if her expected payoff from rebellion exceeds its costs:

$$E\left[\mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{a}{m}\middle| x_i\right] = E\left[\mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{a}{a\Pr(x_j < x^m)}\middle| x_i\right] = \int_{\theta=-\infty}^{\theta^m} \frac{\mathrm{pdf}(\theta|x_i)}{\Pr(x_j < x^m|\theta)} d\theta > c. \tag{8}$$

If a symmetric finite threshold equilibrium exists, the marginal citizen who receives the threshold signal $x_i = x^m$ must be indifferent between rebelling or not. Moreover, as Shadmehr (2019b, Lemma 3) shows, when there is no prior information about θ, the marginal citizen believes that the size of the rebellion is uniformly distributed on $[0, 1]$. But because a rebel only receives the rewards if the rebellion succeeds, the maximum reward size is $\frac{1}{\Pr(x_j<x^m|\theta=\theta^m)}$. Thus, the marginal citizen's expected payoff from rebellion and the indifference condition become

$$\int_{\Pr(x_j<x^m|\theta^m)}^{1} \frac{du}{u} = -\log(\Pr(x_j < x^m|\theta^m)) = c.$$

Combining this with (7) yields $-\log(\theta^m/a) = c$, so that the unique equilibrium regime change threshold is

$$\theta^m = ae^{-c}. \tag{9}$$

Alternatively, let $z(\hat{\theta}) = \Pr(\theta < \hat{\theta}|x_i = x^m)$, with $z^m = \Pr(\theta < \theta^m|x_i = x^m)$. Because there is no prior information about θ: $\Pr(\theta < \theta^m|x_i = x^m) = 1 - \Pr(x_i < x^m|\theta = \theta^m)$. Thus, $1 - z^m = \theta^m/a$. Moreover, the left hand side of the inequality (8) can be re-written in terms of z, so that the indifference condition becomes:

$$\int_{\theta=-\infty}^{\theta^m} \frac{\text{pdf}(\theta|x_i = x^m)}{\Pr(x_j < x^m|\theta)} d\theta = \int_{z=0}^{z^m} \frac{dz}{1-z} = -log(1-z)\Big|_{z=0}^{z^m} = -log(1-z^m) = c. \tag{10}$$

Because $1 - z^m = \theta^m/a$, this shows $\theta^m = ae^{-c}$.

It remains to show that the best response to a monotone strategy is also monotone. Let $\pi(\theta) = \mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{1}{\Pr(x_j<x^m|\theta)} - c$. If $c \in (0,1)$, then $\pi(\theta)$ has a single-crossing property. Because (x,θ) satisfy monotone likelihood ration property, by Karlin's theorem (Shadmehr 2019b, Online Appendix), single-crossing property holds under the integral transformation in (8), and the best response to a monotone strategy is also monotone. This means that the marginal citizen with signal $x_i = x^m$ must be indifferent between rebelling or not. □

Proof of Proposition 4: In the psychological rewards setting, equilibrium conditions are:

$$(a-b)\Pr(x_i < x_r^p|\theta_r^p) = \theta_r^p \quad \text{and} \quad \Pr(\theta < \theta_r^p|x_i = x_r^p) = c.$$

Thus, from proposition 2, $\theta_r^p = (a-b)(1-c)$, which is decreasing in b:

$$\frac{\partial \theta_r^p}{\partial b} = -(1-c) < 0.$$

In the material rewards setting, the belief consistency condition is:

$$(a-b)\Pr(x_i < x_r^m|\theta_r^m) = \theta_r^m.$$

A citizen with signal x_i rebels if and only if:

$$\int_{\theta=-\infty}^{\theta_r^m} \frac{a \;\, \text{pdf}(\theta|x_i)}{(a-b)\Pr(x_j < x_r^m|\theta)} d\theta - c > 0.$$

Let $\pi_r(\theta) = \mathbf{1}_{\{\theta<\theta_r^m\}} \cdot \frac{a}{(a-b)\Pr(x_j<x_r^m|\theta)} - c$. Because $c < 1 \leq \frac{a}{a-b}$, again $\pi_r(\theta)$ has a single-crossing property. Hence, as in the proof of Proposition 2, the best response to a monotone strategy is monotone. Moreover, using the same change of variables as in the proof of Proposition 2, the indifference condition is:

$$c \;=\; \int_{\theta=-\infty}^{\theta_r^m} \frac{a \;\, \text{pdf}(\theta|x_i = x_r^m)}{(a-b)\Pr(x_j < x_r^m|\theta)} d\theta = \frac{a}{a-b}\int_{z=0}^{z_r^m} \frac{dz}{1-z} = -\frac{a}{a-b} log(1-z)\Big|_{z=0}^{z_r^m} = -\frac{a}{a-b} log(1-z_r^m).$$

Moreover, $1 - z_r^m = 1 - \Pr(\theta < \theta_r^m | x_r^m) = \Pr(x_i < x_r^m | \theta_r^m) = \frac{\theta_r^m}{a-b}$. Thus,

$$\frac{a}{a-b}\log(1 - z_r^m) = \frac{a}{a-b}\log\left(\frac{\theta_r^m}{a-b}\right) = -c.$$

Thus,

$$\theta_r^m = (a-b)e^{-c\frac{a-b}{a}},$$

which is decreasing in b:

$$\frac{\partial \theta_r^m}{\partial b} = -e^{-c\frac{a-b}{a}}\left(1 - \frac{a-b}{a}\, c\right) < 0.$$

Next, observe that $\theta^p - \theta_r^p = b(1-c)$ and $\theta^m - \theta_r^m = ae^{-c} - (a-b)e^{-c\frac{a-b}{a}}$. Thus,

$$\frac{\theta^m - \theta_r^m}{\theta^m} \;=\; \frac{ae^{-c} - (a-b)e^{-c\frac{a-b}{a}}}{ae^{-c}} = 1 - \frac{a-b}{a}\, e^{\frac{b}{a}c} < 1 - \frac{a-b}{a} = \frac{b}{a} = \frac{b(1-c)}{a(1-c)} = \frac{\theta^p - \theta_r^p}{\theta^p}. \quad (11)$$

□

Proof of Proposition 5: First, we prove a lemma that characterizes finite cutoff equilibria.

Lemma 1 *$\theta_2^m > \max\{\theta_1^m, 1-a\}$ is an equilibrium regime change threshold of the material rewards setting if and only if is satisfies*

$$\theta_2^m \;=\; \left(1 - a + aF\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m - (1-a)}{a}\right)\right)\right)\; e^{-ac\;F\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m-(1-a)}{a}\right)\right)}, \quad (12)$$

and $\theta_2^p > \max\{\theta_1^p, 1-a\}$ is an equilibrium regime change threshold of the psychological rewards setting if and only if is satisfies

$$\theta_2^p \;=\; 1 - a + a(1-c)F\left(\frac{\theta_2^p - \theta_1^p}{\sigma} + F^{-1}\left(\frac{\theta_2^p - (1-a)}{a}\right)\right). \quad (13)$$

Proof of Lemma 1: We first consider the material rewards setting. From equation (4), the marginal citizen's net expected payoff from rebellion is:

$$E\left[\frac{\mathbf{1}_{\{\theta<\theta_2^m\}}}{1 - a + a\Pr(x_j < x_2^m)}\middle| x_i = x_2^m, \theta > \theta_1^m\right]$$

$$= \int_{\theta_1^m}^{\theta_2^m} \frac{1}{1 - a + a\Pr(x < x_2^m|\theta)}\frac{\mathrm{pdf}(\theta|x_2^m)}{\Pr(\theta > \theta_1^m|x_2^m)}d\theta$$

$$= \int_{\theta_1^m}^{\theta_2^m} \frac{1}{1 - a + aF\left(\frac{x_2^m-\theta}{\sigma}\right)}\frac{f\left(\frac{x_2^m-\theta}{\sigma}\right)g(\theta)}{\int_{\theta_1^m}^{\infty} f\left(\frac{x_m-\theta}{\sigma}\right)g(\theta)d\theta}\, d\theta \quad \text{(let } g(\theta) \text{ be the prior pdf of } \theta\text{)}$$

$$= \frac{\int_{\theta_1^m}^{\theta_2^m} \frac{f\left(\frac{x_2^m-\theta}{\sigma}\right)}{1-a+aF\left(\frac{x_2^m-\theta}{\sigma}\right)}d\theta}{\int_{\theta_1^m}^{\infty} f\left(\frac{x_2^m-\theta}{\sigma}\right)d\theta} \quad (g(\theta) = 1 \text{ for uniform}).$$

Thus,

$$E\left[\frac{\mathbf{1}_{\{\theta<\theta_2^m\}}}{1-a+a\Pr(x_j<x_2^m)}\middle| x_i=x_2^m,\theta>\theta_1^m\right] = \frac{\frac{1}{a}\log\left(\frac{1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}{1-a+aF\left(\frac{x_2^m-\theta_2^m}{\sigma}\right)}\right)}{F\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}$$

$$= \frac{1}{a}\frac{\log\left(\frac{1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}{\theta_2^m}\right)}{F\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}, \tag{14}$$

where the last equality follow from equation (3). Substituting from (14) into (4) yields:

$$\log\left(1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)\right)-\log(\theta_2^m)=acF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right). \tag{15}$$

Substituting x_2^m from (3) into (15) yields:

$$\theta_2^m=\left(1-a+aF\left(\frac{\theta_2^m-\theta_1^m}{\sigma}+F^{-1}\left(\frac{\theta_2^m-(1-a)}{a}\right)\right)\right)\ e^{-ac\ F\left(\frac{\theta_2^m-\theta_1^m}{\sigma}+F^{-1}\left(\frac{\theta_2^m-(1-a)}{a}\right)\right)}. \tag{16}$$

Similarly for the psychological rewards setting, from equation (6), the marginal citizen's net expected payoff from rebellion is:

$$E\left[\mathbf{1}_{\{\theta<\theta_2^p\}}\middle| x_i=x_2^p,\theta>\theta_1^p\right]=\frac{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)-F\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)}{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)}. \tag{17}$$

Thus, any (θ_2^p, x_2^p) that satisfied the following equations constitute an equilibrium.

$$1-a+aF\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)=\theta_2^p \quad\text{and}\quad \frac{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)-F\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)}{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)}=c. \tag{18}$$

Substituting x_2^p from the belief consistency condition into the indifference condition yields:

$$\theta_2^p=1-a+a(1-c)F\left(\frac{\theta_2^p-\theta_1^p}{\sigma}+F^{-1}\left(\frac{\theta_2^p-(1-a)}{a}\right)\right) \tag{19}$$

$\square$

Lemma 2

$$\lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\}=1-a+a(1-c)\quad \textit{and}\quad \lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\}=e^{-ac}.$$

Proof of Lemma 2: When $\theta_2^m > \max\{\theta_1^m, 1-a\}$, we have:

$$\lim_{\sigma \to 0} F\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m - (1-a)}{a}\right)\right) = 1. \tag{20}$$

Thus, the right hand side of (12) approaches e^{-ac}, so that the largest crossing of the 45 degree line approaches e^{-ac}:

$$\lim_{\sigma \to 0} \max\{\theta_2^m(\sigma)\} = e^{-ac}.$$

Similarly,

$$\lim_{\sigma \to 0} \max\{\theta_2^p(\sigma)\} = 1 - a + a(1-c).$$

$\square$

Lemma 2 reflects that when noise in private signals is small, citizens discard their public information. Thus, the informational channel is shut down in the limit. From Lemma 2 and Proposition 2,

$$\Delta^m = \lim_{\sigma \to 0} \max\{\theta_2^m(\sigma)\} - \theta_1^m = e^{-ac} - e^{-c} \quad \text{and} \quad \Delta^p = \lim_{\sigma \to 0} \max\{\theta_2^p(\sigma)\} - \theta_1^p = (1-a)c. \tag{21}$$

$\Delta^m < \Delta^p$ if and only if $\frac{e^{-c} - e^{-ac}}{c - ac} > -1$, which is true because e^{-x} is strictly decreasing and convex with $\frac{de^{-x}}{dx}\big|_{x=0} = -1$. $\square$

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Online Appendices

A Mixed Motivations

We have compared the two extreme kinds of rewards, purely material or purely psychological. But as Kennedy (1999) aptly puts in his study of "the rumbles of discontent" during the Great Depression, people "can subsist on solely spiritual nourishment little longer than they can live on bread alone" (218).[10] We now analyze movements in which incentives to rebel are a combination of material and psychological rewards. We generalize our payoffs in Figure 1 by adding a parameter $\bar{m} \in [0,1]$ that generates our purely material rewards setting in one extreme ($\overline{m} = 0$) and our purely psychological rewards setting in the other extreme ($\overline{m} = 1$). Figure 2 shows the payoffs—we normalize the population size to 1.

	outcome	
	$m \geq \theta$	$m < \theta$
rebel	$\frac{1}{\overline{m}}(\mathbf{1}_{\{m \leq \overline{m}\}} + \mathbf{1}_{\{m \geq \overline{m}\}} \cdot \frac{\overline{m}}{m}) - c$	$-c$
not rebel	0	0

Figure 2: Payoffs combining material and psychological motivations.

Proposition 6 characterizes the unique equilibrium. Figure 3 illustrates the result.

Proposition 6 *Let θ^* be the equilibrium regime change threshold in the setting with mixed motivations. Then,*

$$\theta^* = \begin{cases} e^{-c} & ; \overline{m} \leq e^{-c} \\ \\ \overline{m}\ (1 - c - \log(\overline{m})) & ; \overline{m} \geq e^{-c}. \end{cases}$$

Moreover, $\theta^m > \theta^*(\overline{m}) > \theta^p$ for $\overline{m} \in (0,1)$, with $\lim_{\overline{m} \to 0} \theta^* = \theta^m$ and $\lim_{\overline{m} \to 1} \theta^* = \theta^p$.

Proof of Proposition 6: The net payoff from rebelling versus not is:

$$\frac{1}{\overline{m}} \left(\mathbf{1}_{\{\theta < m, m \leq \overline{m}\}} + \mathbf{1}_{\{\theta < m, m \geq \overline{m}\}} \cdot \frac{\overline{m}}{m} \right) - c \tag{22}$$

[10] Kennedy, David M. 1999. *Freedom from Fear: The American People in Depression and War, 1929-1945.* New York: Oxford University Press.

As in the pure material rewards setting, this net payoff is non-monotone in the fraction of rebels m. It jumps up at $m = \theta$ (the threshold at which regime change succeeds), but then falls, weakly is some range and strictly in others, as more citizens join the movement.

As before, given a value of θ, the fraction of rebels is $m(\theta) = \Pr(x < x^*|\theta)$, and $\Pr(x_i < x^*|\theta^*) = \theta^*$. Moreover, $m(\theta) < \overline{m}$ if and only if $\theta > \bar{\theta}$, where $\Pr(x_i < x^*|\bar{\theta}) = \overline{m}$. Then, the net expected payoff from rebellion versus not is:

$$\int_{\theta=-\infty}^{\infty} \frac{1}{\overline{m}} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)} \right) \text{pdf}(\theta|x_i) - c. \tag{23}$$

As before, if $c < \min\{1, 1/\overline{m}\} = 1$, we can invoke the Karlin's Theorem to conclude to that the best response to a monotone strategy is also monotone. The indifference condition is:

$$\int_{\theta=-\infty}^{\infty} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)} \right) \text{pdf}(\theta|x_i = x^*) = \overline{m}\ c. \tag{24}$$

First, suppose $\bar{\theta} > \theta^*$. Then,

$$\int_{\theta=-\infty}^{\infty} \mathbf{1}_{\{\theta<\theta^*\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)}\ \text{pdf}(\theta|x_i = x^*) = \overline{m}\ c. \tag{25}$$

Thus,

$$\theta^* < \bar{\theta} \ \Rightarrow\ \theta^* = e^{-c}, \tag{26}$$

where we recognize that $\bar{\theta}$ is endogenous and depends on x^*. However, recall that $\Pr(x < x^*|\bar{\theta}) = \overline{m}$ and $\Pr(x < x^*|\theta^*) = \theta^*$. Thus, $\theta^* < \bar{\theta}$ is equivalent to $\theta^* > \overline{m}$. Given (26), $\theta^* > \overline{m}$ is equivalent to: $-c > \log(\overline{m})$.

Next, suppose $\bar{\theta} < \theta^*$, i.e., $\theta^* < \overline{m}$. Then,

$$\begin{aligned} \overline{m}\ c &= \int_{\theta=-\infty}^{\bar{\theta}} \frac{\overline{m}}{\Pr(x_i < x^*|\theta)}\ \text{pdf}(\theta|x_i = x^*)d\theta + \int_{\bar{\theta}}^{\theta^*} \text{pdf}(\theta|x_i = x^*)d\theta \\ &= -\overline{m}\ \log(1 - \Pr(\theta < \bar{\theta}|x_i = x^*)) + \Pr(\theta < \theta^*|x_i = x^*) - \Pr(\theta < \bar{\theta}|x_i = x^*). \end{aligned} \tag{27}$$

Substituting for $\Pr(x_i < x^*|\bar{\theta}) = \overline{m} = 1 - \Pr(\theta < \bar{\theta}|x_i = x^*)$ and $\Pr(\theta < \theta^*|x_i = x^*) = 1 - \theta^*$ yields $-\overline{m}\ \log(\overline{m}) + \overline{m} - \theta^* = \overline{m}\ c$, i.e.,

$$\theta^* = \overline{m}\ (1 - \log(\overline{m})) - \overline{m}\ c. \tag{28}$$

Thus, $\theta^* < \overline{m}$ if and only if $-c < \log(\overline{m})$.

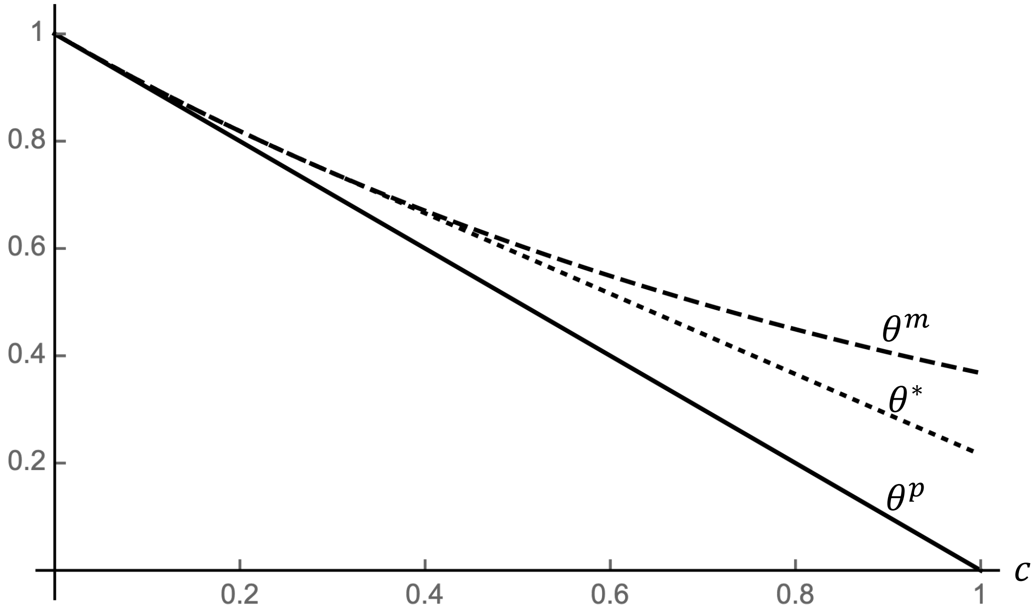

Figure 3: The equilibrium regime change threshold for the settings with pure psychological rewards (solid line, θ^p), pure material rewards (dashed curve, θ^m), and a mix of psychological and material rewards (dotted curve, θ^*). Parameters: $\overline{m} = 0.75$.

Combining this results yield:

$$\theta^* = \begin{cases} e^{-c} & ; c \leq -\log(\overline{m}) \\ \\ \overline{m}\,(1 - c - \log(\overline{m})) & ; c \geq -\log(\overline{m}) \end{cases} \tag{29}$$

We observe that

$$\left.\frac{\theta^*(c)}{dc}\right|_{c=-\log(\overline{m})} = -\overline{m}.$$

□

Proposition 6 and Figure 3 show that when motivations are a mix of psychological and material, the effects of targeted repression, indiscriminate repression, and early failure lie in between those effects in the settings with material and psychological rewards analyzed earlier. The key intuition comes from thinking about the extent to which rewards are rival. In the pure material rewards setting, rewards are entirely rival. In the pure psychological rewards setting, rewards are entirely non-rival. In this mixed setting, we can think of some portion of the rewards as being rival and another portion being non-rival.

Interestingly, this points to a different interpretation of this version of the model, where we interpret the rewards as material, but imperfectly divisible, such as promises to hold future government office. Suppose there are a total of $\overline{m}$ offices available. If the rebellion is small, $m < \overline{m}$, and succeeds, each participant in the rebellion gets an office. But there are too many offices for the rebels to fill all of them. So some offices must be left in the hands of their previous holders. (Think of a small rebel group not fully purging the bureaucracy after taking control of the state.) However, if the rebellion is large, $m > \overline{m}$, there are not enough offices to go around and the congestion externality returns. So, if the number of participants is smaller than the number of offices, an increase in participation in the rebellion does not diminish the rewards an individual enjoys should they success. For example, if 1000 offices are available, whether 600 or 800 citizens rebel, there are enough offices for each to get one. This feature shares the non-rival aspect of the psychological rewards setting. However, if the number of participants exceed 1000, further increases in the number of participants reduces the chances that each rebel receives an office upon success because there will not be enough government offices to go around. This feature shares the rival aspect of the material rewards setting.

The real world, of course, is not so clear cut. More offices can be created and responsibilities may be shared. However, the insight that government offices tend to be more discrete than, for example, cash, diamonds, or land remains true. As such, in settings where such offices are the main reward of victory, the effect of repression on the rebel movement falls between the effects in settings with pure (continuous) material rewards and settings with psychological rewards.

B Relaxing Informational Assumptions

In the text, we focused on the case in which players share a prior that θ is distributed uniformly on $\mathbb{R}$ (improper prior). With a smooth (proper) prior, the same results obtain in the limit when the information content of the prior becomes vanishingly small, e.g., $\theta \sim N(\mu, \sigma_0)$ when σ_0 becomes unboundedly large. Here, we show that the same results also obtain for any smooth prior in the limit when the noise becomes vanishingly small ($\sigma \to 0$). We then provide numerical examples for a standard normal prior for both the case of a uniform distribution of noise and a standard normal distribution of noise. Finally, we provide additional numerical examples for the effect of a public signal about the strength of the regime (θ) in both settings with psychological and material rewards.

Consider the setting in the text, but suppose $\theta \sim G$, where $G(\cdot)$ is smooth and $G(\theta) \in (0,1)$ for all $\theta \in \mathbb{R}$. Let $g(\cdot)$ be the corresponding pdf. We begin by proving that $\theta^m > \theta^p$. From the belief consistency condition, $x^j = \theta^j + \sigma F^{-1}(\theta^j/a)$, $j \in \{p, m\}$. Because the right hand side is increasing in θ^j, it is invertible. Define $\Omega(\cdot)$, so that $\theta^j = \Omega(x^j)$. Thus, the indifference conditions can be written as:

$$c = \int_{-\infty}^{\Omega(x^m)} \frac{\text{pdf}(\theta|x^m)}{F\left(\frac{x^m-\theta}{\sigma}\right)} d\theta = \int_{-\infty}^{\Omega(x^p)} \text{pdf}(\theta|x^p) d\theta. \tag{30}$$

Because $F(\cdot)$ in the denominator is less that 1, we have:

$$c < \int_{-\infty}^{\Omega(x^p)} \frac{\text{pdf}(\theta|x^p)}{F\left(\frac{x^p-\theta}{\sigma}\right)} d\theta.$$

Thus, $x^m = x^p$ (and hence $\theta^m = \theta^p$) cannot be part of the equilibrium in the material rewards setting. x^m (and hence θ^m) must adjust to restore the equilibrium. In the stable equilibrium, they must increase, so that higher costs c imply higher likelihoods of regime change. Thus, we have:

Proposition 7 *In a stable equilibrium of the material rewards setting, $\theta^m > \theta^p$.*

To further characterize the equilibrium regime change thresholds, we provide analytical results when the noise is very small ($\sigma \to 0$) and numeral results when noise is larger.

B.1 Analytical Results for Vanishingly Small Noise

Lemma 3 *θ^j, $j \in \{p, m\}$, is an equilibrium regime change threshold if it satisfies the following equation:*

$$c = \Gamma_j(\theta^j;\sigma) \equiv \frac{\int_{F^{-1}(\theta^j/a)}^{\infty} f(z)\ g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))\ \frac{1}{\mathbf{1}_{\{j=m\}}F(z)+\mathbf{1}_{\{j=p\}}} dz}{\int_{-\infty}^{\infty} f(z)\ g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))\ dz},$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function.

Proof of Lemma 3: The belief consistency condition is:

$$\frac{\theta^j}{a} = F\left(\frac{x^j - \theta^j}{\sigma}\right). \tag{31}$$

The indifference condition is:

$$\begin{aligned}
c &= \int_{\theta=-\infty}^{\theta^j} \mathrm{pdf}(\theta|x^j)\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(\frac{x^m-\theta}{\sigma}\right) + \mathbf{1}_{\{j=p\}}}\ d\theta \\
&= \int_{\theta=-\infty}^{\theta^j} \frac{f\left(\frac{x^j-\theta}{\sigma}\right) g(\theta)}{\int_{\theta=-\infty}^{\infty} f\left(\frac{x^j-\theta}{\sigma}\right) g(\theta) d\theta}\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(\frac{x^m-\theta}{\sigma}\right) + \mathbf{1}_{\{j=p\}}}\ d\theta \\
&= \int_{z=z^j}^{\infty} \frac{f\left(z\right) g(x^j - \sigma z)}{\int_{z=-\infty}^{\infty} f\left(z\right) g(x^j - \sigma z) dz}\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(z\right) + \mathbf{1}_{\{j=p\}}}\ dz \\
&= \int_{z=F^{-1}(\theta^j/a)}^{\infty} \frac{f\left(z\right) g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))}{\int_{z=-\infty}^{\infty} f\left(z\right) g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z)) dz}\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(z\right) + \mathbf{1}_{\{j=p\}}}\ dz \quad \text{(from (31))},
\end{aligned}$$

where, in the third equality, we did a change of variables from θ to $z = \frac{x^j-\theta}{\sigma}$, with $z^j = \frac{x^j-\theta^j}{\sigma}$. $\square$

In the limit when $\sigma \to 0$, the terms involving $g(\cdot)$ in Lemma 3 will cancel, and θ^j simplifies to those in Proposition 2 in the text with improper uniform prior.

Proposition 8 *In the limit when the noise become vanishingly small ($\sigma \to 0$) we have:*

$$\lim_{\sigma\to 0} \Gamma_j(\theta^j;\sigma) = \begin{cases} 1 - \theta^p/a & ; j = p \\ -\log(\theta^m/a) & ; j = m, \end{cases}$$

so that $\theta^p = a(1-c)$ and $\theta^m = ae^{-c}$.

Proof of Proposition 8: From Lemma 3,

$$\lim_{\sigma\to 0} \Gamma_j(\theta^j;\sigma) = \int_{F^{-1}(\theta^j/a)}^{\infty} f(z)\ \frac{1}{\mathbf{1}_{\{j=m\}}F(z) + \mathbf{1}_{\{j=p\}}} dz = \begin{cases} 1 - F(F^{-1}(\theta^p/a)) & ; j = p \\ \log(1) - \log(F(F^{-1}(\theta^m/a))) & ; j = m. \end{cases}$$

$\square$

B.2 An Example: Uniform Noise

We next provide a simple example to demonstrate a special case of this general result. Suppose $F = U[-1, 1]$. Then,

$$\text{pdf}(\theta|x_i) = \frac{\text{pdf}(x_i|\theta)g(\theta)}{\int_{-\infty}^{\infty} \text{pdf}(x_i|\theta)g(\theta)d\theta} = \begin{cases} \frac{\frac{1}{2\sigma}g(\theta)}{\int_{x_i-\sigma}^{x_i+\sigma}\frac{1}{2\sigma}g(\theta)d\theta} = \frac{g(\theta)}{G(x_i+\sigma)-G(x_i-\sigma)} & ;\theta - \sigma \leq x_i \leq \theta + \sigma \\ 0 & ; otherwise. \end{cases} \tag{32}$$

Thus, for a given $\hat{\theta}$ and $\hat{x}$,

$$\Pr(\theta < \hat{\theta}|x_i = \hat{x}) = \begin{cases} 0 & ;\hat{\theta} \leq \hat{x} - \sigma \\ \frac{G(\hat{\theta})-G(\hat{x}-\sigma)}{G(\hat{x}+\sigma)-G(\hat{x}-\sigma)} & ;\hat{x} - \sigma \leq \hat{\theta} \leq \hat{x} + \sigma \\ 1 & ;\hat{x} + \sigma \leq \hat{\theta}. \end{cases} \tag{33}$$

Similarly,

$$\Pr(x_i < \hat{x}|\theta = \hat{\theta}) = \begin{cases} 1 & ;\hat{\theta} \leq \hat{x} - \sigma \\ \frac{\hat{x}-(\hat{\theta}-\sigma)}{2\sigma} & ;\hat{x} - \sigma \leq \hat{\theta} \leq \hat{x} + \sigma \\ 0 & ;\hat{x} + \sigma \leq \hat{\theta}. \end{cases} \tag{34}$$

Lemma 4 *For any $\hat{x}$ and $\hat{\theta}$, we have:*

$$\lim_{\sigma \to 0} \Pr(\theta < \hat{\theta}|x_i = \hat{x}) = 1 - \lim_{\sigma \to 0} \Pr(x_i < \hat{x}|\theta = \hat{\theta}).$$

Proof of Lemma 4: From equations (33) and (34), the result is immediate for the cases of $\hat{\theta} \leq \hat{x} - \sigma$ and $\hat{x} + \sigma \leq \hat{\theta}$. For completeness, consider $\hat{x} - \sigma \leq \hat{\theta} \leq \hat{x} + \sigma$ and equation (33). Using a Taylor's expansion, in the limit $\sigma \to 0$, we have:

$$\begin{aligned} G(\hat{\theta}) - G(\hat{x} - \sigma) &= G(\hat{x} + (\hat{\theta} - \hat{x})) - G(\hat{x} - \sigma) = G(\hat{x}) + g(\hat{x})(\hat{\theta} - \hat{x}) - (G(\hat{x}) - g(\hat{x})\sigma) \\ &= g(\hat{x})(\hat{\theta} - \hat{x} + \sigma). \end{aligned} \tag{35}$$

Similarly,

$$G(\hat{x} + \sigma) - G(\hat{x} - \sigma) = G(\hat{x}) + g(\hat{x})\sigma - (G(\hat{x}) - g(\hat{x})\sigma) = g(\hat{x})2\sigma. \tag{36}$$

Combining equations (35) and (36), for $\hat{x} - \sigma \leq \hat{\theta} \leq \hat{x} + \sigma$, we have:

$$\lim_{\sigma \to 0} \frac{G(\hat{\theta}) - G(\hat{x} - \sigma)}{G(\hat{x} + \sigma) - G(\hat{x} - \sigma)} = \frac{g(\hat{x})(\hat{\theta} - \hat{x} + \sigma)}{g(\hat{x})2\sigma} = 1 - \frac{\hat{x} - (\hat{\theta} - \sigma)}{2\sigma} = 1 - \Pr(x_i < \hat{x}|\theta = \hat{\theta}).$$

□

As shown in Shadmehr (2019a,b), Lemma 4 is the statistical property that delivers the uniform beliefs property, which, in turn, delivers the result in Proposition 2 in the text.

B.3 Numerical Simulations for Larger Noise

We have established the results analytically in the asymptotic cases of vanishingly small noise and no prior information (about θ). The results also hold when the noise is sufficiently small, or there is sufficiently little common knowledge, e.g., the prior is $N(\mu, \sigma_0)$ and σ_0 is sufficiently large. We now provide numerical simulations to show that our results are not limited to the cases of very small noise or very little common knowledge, leaving to future research a fuller characterization of the interactions between information and motivation in contentious settings.

First, we continue the example above by providing a numerical example for the case of standard normal prior distribution and uniform noise distribution: $G = N(0, 1)$, $F = U[-1, 1]$, and $\sigma = 1$. From equation (34), the belief consistency condition can be written as:

$$\frac{\theta^j}{a} = \min\{1, \max\{0, \frac{1}{2} + \frac{x^j - \theta^j}{2\sigma}\}\},\ j \in \{p, m\}.$$

Because $\theta^j/a \in (0, 1)$, we have:

$$\theta^j = \frac{x^j + \sigma}{\frac{2\sigma}{a} + 1},\ j \in \{p, m\}. \tag{37}$$

From equation (32), the indifference condition can be written as:

$$\begin{aligned} c &= \int_{-\infty}^{\theta^j} \frac{g(\theta) \cdot \mathbf{1}_{\{x^j - \sigma \leq \theta \leq x^j + \sigma\}}}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \Pr(x_i \leq x^j | \theta) + \mathbf{1}_{\{j=p\}}} d\theta \\ &= \int_{x^j - \sigma}^{\theta^j} \frac{g(\theta)}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \left(\frac{1}{2} + \frac{x^m - \theta}{2\sigma}\right) + \mathbf{1}_{\{j=p\}}} d\theta,\ j \in \{p, m\}. \end{aligned} \tag{38}$$

Substituting from (37) into (38) yields:

$$c = R^j(x^j) \equiv \int_{x^j - \sigma}^{\frac{x^j + \sigma}{\frac{2\sigma}{a} + 1}} \frac{g(\theta)}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \left(\frac{1}{2} + \frac{x^m - \theta}{2\sigma}\right) + \mathbf{1}_{\{j=p\}}} d\theta,\ j \in \{p, m\}. \tag{39}$$

To demonstrate, suppose $G = N(0, 1)$, and $\sigma = a = 1$, so that $\theta^j = \frac{x^j + 1}{3}$. Figure 4 shows $R^j(x^j)$, $j \in \{p, m\}$. Both $R^m(x)$ and $R^p(x)$ are decreasing, so that raising the costs (c) reduces the equilibrium threshold. Moreover, when c approaches 0, both x^m and x^p approach 2, implying that θ^m and θ^p approach 1. When, instead, c approaches 1, x^p approaches -1, so that

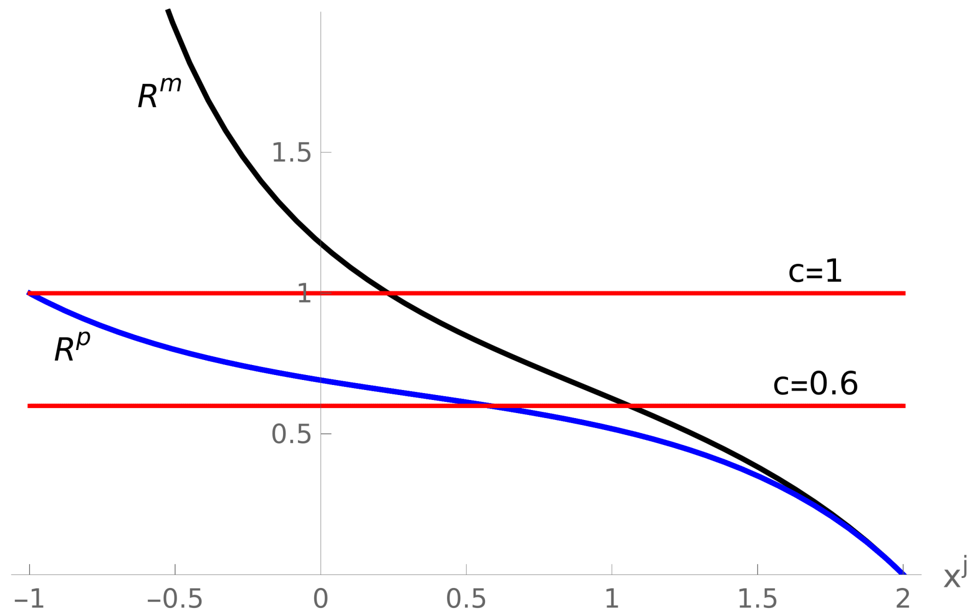

Figure 4: An example with $G = N(0,1)$, $F = U[-1,1]$, and $\sigma = a = 1$. From equation (39), the equilibrium threshold satisfies $R^j(x^j) = c$, $j \in \{p, m\}$. Note that an increase in c causes a sharper reduction in x^p than in x^m.

θ^p approaches 0. In contrast, x^m and hence θ^m both remain positive as in our model with no prior information about θ. Critically, $R^m(x)$ changes faster with x, so that as increase in costs c causes a smaller reduction in x^m than in x^p. Equation (39) shows the additional term $\frac{1}{2} + \frac{x^m - \theta}{2\sigma}$ in the denominator for the material rewards settings. When c increases, in any stable equilibrium, the equilibrium threshold x^j must fall so restore the indifference condition—citizens become less likely to revolt. In our example, the presence of $\frac{1}{2} + \frac{x^m - \theta}{2\sigma}$ in the denominator causes x^m to fall by less. That is, the same reduction in x^j has a larger effect in restoring the indifference condition and the equilibrium in the material rewards setting.

Next, we provide a numerical example when both the prior and the noise have the standard normal distribution: $G = N(0, \sigma_0)$, $F = N(0,1)$, and $\sigma_0 = \sigma = a = 1$. The belief consistency condition is:

$$\theta^j = \Phi\left(\frac{x^j - \theta^j}{\sigma}\right), \text{ so that } x^j = \theta^j + \sigma\Phi^{-1}(\theta^j). \tag{40}$$

The indifference conditions are:

$$c = \Phi\left(\frac{\theta^p - bx^p}{\sqrt{b\sigma^2}}\right) = \int_{-\infty}^{\theta^m} \frac{\frac{1}{\sqrt{b\sigma^2}}\phi\left(\frac{\theta - bx^m}{\sqrt{b\sigma^2}}\right)}{\Phi\left(\frac{x^m - \theta}{\sigma}\right)} \, d\theta, \text{ where } b = \frac{\sigma_0^2}{\sigma^2 + \sigma_0^2}. \tag{41}$$

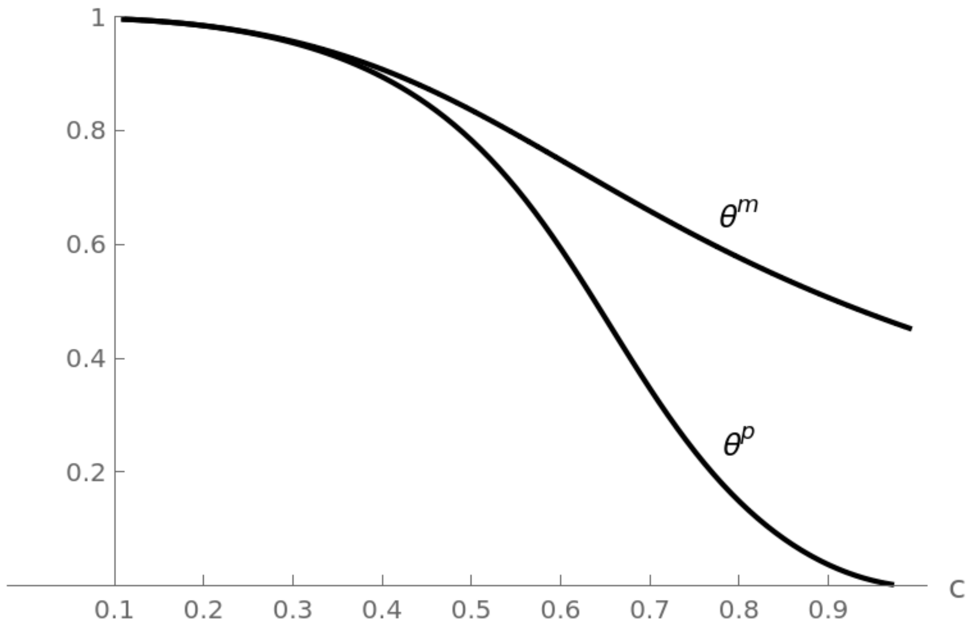

Figure 5: The equilibrium regime change threshold for the psychological rewards setting (θ^p), and material rewards setting (θ^m) when $\theta \sim N(0,1)$, $\epsilon_i \sim iidN(0,1)$, and $\sigma = a = 1$.

Substituting from (40) into (41) yields:

$$c = \Phi\left(\frac{(1-b)\theta^p - b\sigma\Phi^{-1}(\theta^p)}{\sqrt{b\sigma^2}}\right) = \int_{-\infty}^{\theta^m} \frac{\frac{1}{\sqrt{b\sigma^2}}\phi\left(\frac{\theta - b\theta^m - b\sigma\Phi^{-1}(\theta^m)}{\sqrt{b\sigma^2}}\right)}{\Phi\left(\frac{\theta^m + \sigma\Phi^{-1}(\theta^m) - \theta}{\sigma}\right)}\, d\theta. \tag{42}$$

Based on (42), Figure 5 demonstrates the equilibrium regime change thresholds θ^p and θ^m as functions of costs c when $\sigma_0 = \sigma = a = 1$.

B.4 Public Signal

To further highlight the logic behind our results, we also compare the effect of public signals about the regime's strength on the equilibrium regime change threshold in the psychological and material rewards settings. There is a link between this analysis and our discussion of a general prior. Suppose players share an improper uniform prior about θ as in the paper, but receive a noisy public signal p about θ in the form of $p = \theta + \sigma_p \nu$, where $\nu \sim H$. This setting is equivalent to players having a (proper) prior with mean p. For example, if $\nu \sim N(0,1)$ and $p = 1$, players will share a prior that $\theta \sim N(1, \sigma_p)$. In particular, beginning from no prior information about θ, the public signal will generates some common knowledge about θ. We now investigate the effect of a higher public signal in both settings. A higher public signal generates

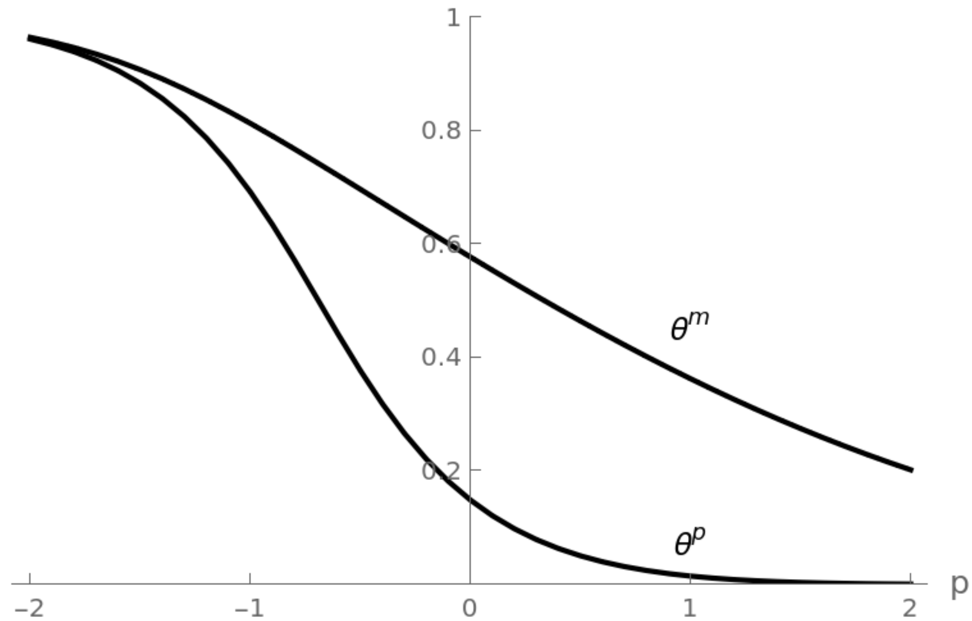

Figure 6: The equilibrium regime change threshold for the psychological rewards setting (θ^p), and material rewards setting (θ^m) as a function of a public signal $p = \theta + \nu$. Parameters: $G = N(0,1)$, $\nu \sim N(0,1)$, $F = N(0,1)$, $\sigma = a = 1$.

common knowledge that the regime is stronger, and hence less likely to collapse. As a result, both θ^p and θ^m and the likelihood of regime change fall. Figure 6 illustrates the equilibrium regime change thresholds for different values of p for the came of Normal noise: $\nu \sim N(0,1)$, $F = N(0,1)$, $\sigma = a = 1$. As expected $\theta^m > \theta^p$. Moreover, as long as θ^p/θ^m is not too small, the marginal effect of a higher p is lower in the material rewards settings. The logic is the same as before: all else equal, a higher p reduces the citizens' incentives to revolt, and hence the likelihood of regime change. But, in the material rewards setting, a smaller number of revolutionaries makes the rewards of a successful regime change larger, thereby partially canceling the first effect. All else equal is important. An p increases, the probably of regime change in the psychological rewards setting falls to almost 0. Beginning from such a low probability, the marginal effect of a higher p then becomes very small. Thus, for the right comparison, one must compare the slopes of θ^p and θ^m when the levels are about the same ($\theta^p \approx \theta^m$). Now, it is clear that the marginal effect of a higher p is much smaller at in the material rewards setting.

C Normalization of Material Rewards

In the text, we normalized material rewards to $\frac{a}{m}$, so that if the rebellion succeeds, the total available reward in both settings is a. We now explore the robustness of our results by using more general payoffs. In particular, we assume that material rewards are $\frac{k\times a}{m}$, for some $k>0$. Our analysis in the text corresponds to $k=1$. Given the payoff structure represented in Figure 1, this change is equivalent to normalizing the costs in the material rewards setting from $c\in(0,1)$ to $c/k\in(0,1)$. Then, the equilibrium regime change thresholds in Proposition 2 become: $\theta^p=a(1-c)$ (as before) and $\theta^m=ae^{-c/k}$ (new). Thus, $\frac{d\theta^p}{dc}=-a$ and $\frac{d\theta^m}{dc}=-\frac{a}{k}e^{-c/k}$, so that $\frac{d\theta^p}{dc}<\frac{d\theta^m}{dc}$ if and only if $e^{-c/k}<k$, i.e., $-k\log(k)<c$. When $k\geq 1$, the left hand side is non-positive, and the inequality holds for all $c>0$. When $k<1$, this inequality holds at the upper bound of $c=k<1$ if and only if $-k\log(k)<k$, i.e., $k>1/e\approx 0.37$. Then, there will be a threshold $\hat{c}\in(0,k)$ such that $-k\log(k)<c$ if and only if $c>\hat{c}(k)$. To summarize:

Proposition 9 $\frac{d\theta^p}{dc}<\frac{d\theta^m}{dc}$ *if and only if either* $k\geq 1$, *or* $k>1/e$ *and* $c>\hat{c}(k)$, *where* $\hat{c}\in(0,k)$.

Proposition 5 has a similar analogue. In the proof of Proposition 5, observe that changing c to c/k in the material rewards setting will change (21) to:

$$\Delta^m=\lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\}-\theta_1^m=e^{-ac/k}-e^{-c/k}\quad\text{and}\quad\Delta^p=\lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\}-\theta_1^p=(1-a)c.$$

Thus, $\Delta^m<\Delta^p$ if and only if $\frac{e^{-c/k}-e^{-ac/k}}{c-ac}>-1$, i.e., $\frac{e^{-c/k}-e^{-ac/k}}{c/k-ac/k}>-k$. Now, let $d=c/k\in(0,1)$ and observe that d can change independently of k. Thus, $\Delta^m<\Delta^p$ if and only if $\frac{e^{-d}-e^{-ad}}{d-ad}>-k$, for $d\in(0,1)$. Because e^{-x} is strictly decreasing and convex with $\frac{de^{-x}}{dx}\Big|_{x=0}=-1$, this inequality holds for all $k\geq 1$. When $k<1$, as long as $k>1/e$, there exists $a,c/k\in(0,1)$ such that $\Delta^m<\Delta^p$. To see this, observe that $\frac{de^{-x}}{dx}\Big|_{x=1}=-1/e$. Thus, we have:

Proposition 10 *Suppose the noise in the second period's private signals becomes vanishingly small, and we focus on the largest equilibrium. Conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting if (i)* $k\geq 1$, *or (ii)* $k>1/e$ *and* a *and* c *are sufficiently large.*

In the proof of Proposition 4, observe that changing c to c/k in the material rewards setting will change (11) to:

$$\frac{\theta^m-\theta^m_r}{\theta^m}=\frac{ae^{-c/k}-(a-b)e^{-\frac{a-b}{a}\frac{c}{k}}}{ae^{-c/k}}=1-\frac{a-b}{a}\,e^{\frac{b}{a}\frac{c}{k}}<1-\frac{a-b}{a}=\frac{b}{a}=\frac{\theta^p-\theta^p_r}{\theta^p}.$$

The inequality holds because $e^{\frac{b}{a}\frac{c}{k}}>1$, and hence Proposition 4 does not change.